# Theory

2.1 Measurement of thermal resistance using a DC bridge

A DC balanced bridge (the so-called Wheatstone bridge) is an electrical circuit for measuring an unknown electrical resistance by balancing the two legs of a bridge circuit, with one leg containing the unknown component. The circuit schematic of a DC balanced bridge is shown in Figure 1.

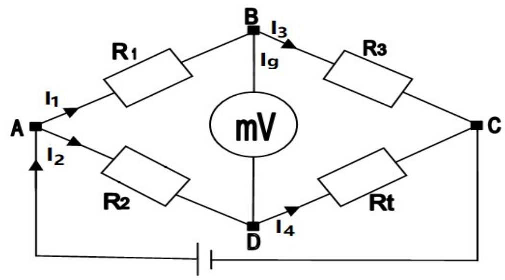


Figure 1 Circuit schematic of a DC bridge

In Figure 1, $R_t$ is the unknown resistor to be measured; $R_1$, $R_2$ and $R_3$ are resistors of known resistance. Adjustable resistor $R_2$ is adjusted to a value when the bridge is "balanced" and no current flows through the millivolt meter (mV). Under such case, the voltage between the two midpoints (B and D) is zero. Hence, the resistance ratio in the known leg ($R_2/R_1$) is equal to that in the unknown leg ($R_t/R_3$) as,

$$\frac{R_1}{R_2} = \frac{R_3}{R_t} \Rightarrow R_t = \frac{R_1}{R_2} R_3 \tag{1}$$

If $R_2 = R_1$, we have $R_t = R_3$.

2.2 Measurement of thermal resistance using a constant current

Figure 2 is the circuit schematic of measuring thermal resistance using a constant current.

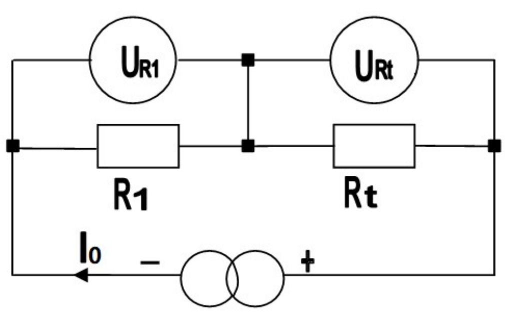


Figure 2 Circuit schematic of measuring thermal resistance using a constant current

In Figure 2, the power supply is a constant current source $I_0$; $R_1$ is a fixed resistor of known value; $R_t$ is a thermal resistor; $U_{R1}$ is the voltage drop on $R_1$; $U_{Rt}$ is the voltage drop on $R_t$. $U_{R1}$ is used to monitor the current of the circuit from which the resistance of $R$t can be derived using the voltage $U_{Rt}$ on the thermal resistor.

When the circuit current is $I_0$ and the temperature is t, the thermal resistor $R_t$ is:

$$R_t = \frac{U_{Rt}}{I_O} = \frac{R_1 U_{Rt}}{U_{R1}} \quad (2)$$

2.3 Temperature sensor Pt100 of platinum resistor

Pt100 platinum resistor is a temperature sensor that utilizes the resistance variation of platinum metal with temperature. Platinum possesses stable physical and chemical characteristics, strong antioxidant ability, good replication, and easy production. Hence, platinum resistors are widely used for precise measurement of temperature in industry. However, the disadvantages are their high cost, small temperature coefficient, and vulnerability to magnetic field perturbation.

According to IEC standards, the range of temperature measurement using a platinum resistance is -200 - 650 °C. Pt100 platinum resistance sensor has a resistance of 100 ohms at 0 °C (i.e. 32 °F). The relationship between resistance $R_t$ and temperature $t$ can be found in the table of Pt100 sensor (Appendix 1), and can be expressed as follows:

$$R_t = R_0\left[1 + At + Bt^2 + C(t - 100°C)t^3\right]$$ , for temperature $t$: -200 ~ 0 °C. (3)

and $$R_t = R_0(1 + At + Bt^2)$$ , for temperature $t$: 0 ~ 650 °C. (4)

In (3) and (4), $R_t$ and $R_0$ are respectively the resistance at temperature $t$ and temperature 0 °C. $A$, $B$ and $C$ are temperature coefficients. For commonly used industrial platinum resistances, we have $A$=3.90802×10$^{-3}$/°C, $B$=-5.80195×10$^{-7}$/°C$^2$, $C$=-4.27350×10$^{-12}$/°C$^3$. In between 0 ~ 100 °C, $R_t$ can be approximately expressed in linear as:

$$R_t = R_0(1 + A_1 t) , \quad (5)$$

where $A_1$ is the temperature coefficient and approximately 3.85×10$^{-3}$/°C. $Rt$ = 100 ohm at $t$ = 0 °C and $Rt$ =138.5 ohm at $t$ = 100 °C.

2.4 Temperature sensor of semiconductor thermistor (NTC1K)

A semiconductor thermistor measures temperature by using the characteristics of resistance variation with temperature change. According to the resistance increasing or reducing with the increase of temperature, semiconductor thermistors are classified into NTC type (negative temperature coefficient), PTC type (positive temperature coefficient) and CTC type (critical temperature). Semiconductor thermistors have properties of large resistivity and large temperature coefficient, but also drawbacks of large nonlinearity and poor stability. Therefore they are usually only applicable to those general sceneries of high accuracy of temperature measurement unnecessary. Characteristic curves of the above three types of thermistors are shown in Figure 3.

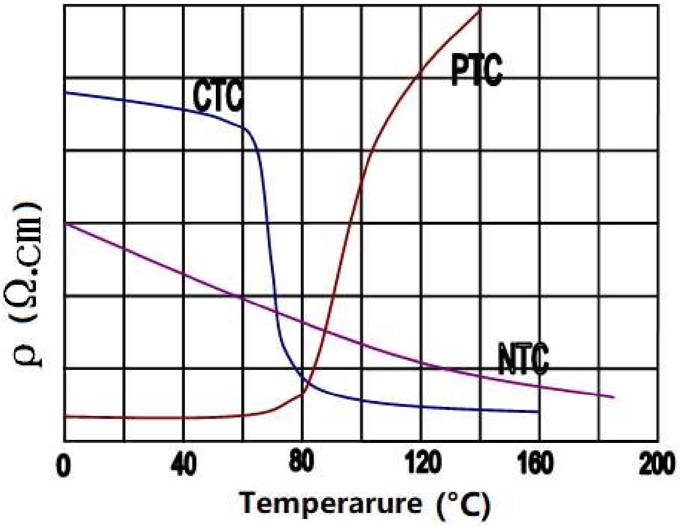

Figure 3 Characteristic curves of three types of thermistors

In a certain temperature range (less than 450 °C), the resistance $R_t$ of a thermistor has a relationship with temperature $T$ as follows:

$$R_T = R_0 e^{B(\frac{1}{T}-\frac{1}{T_0})} \quad (6)$$

where $R_t$ and $R_0$ are respectively the resistance values at temperature $T(K)$ and $T_0(K)$ with $K$ the thermodynamic temperature unit Kelvin. For a certain thermistor, $B$ is a constant and usually *2000 ~ 6000 K*.

Take a logarithm operation on both sides of the above formula, we get:

$$\ln R_T = B(\frac{1}{T}-\frac{1}{T_0}) + \ln R_0 \quad (7)$$

From (7), we know $\ln R_T$ and $1/T$ is in linear relationship. By plotting $\ln R_T$ - ($1/T$) curve and doing a straight line fitting, the slope of the line, i.e. constant $B$, can be acquired.

2.5 Temperature sensor of voltage-mode IC (LM35)

The temperature sensor LM35 is packed following industrial standard $T_0$-92 with an accuracy usually ± 0.5 °C. Since its output is a voltage signal of good linearity, it is easy to build a precision digital temperature measurement system as long as a DC voltage source and a digital voltmeter are supplied. Internal laser calibration ensures high accuracy and consistency without external calibration. The temperature coefficient of output voltage is $K_V$ = 10.0 mV/°C. Figure 4 shows the schematic of LM35. By measuring its output voltage $V_0$, the measured temperature can be calculated using the following formula:

$$t\,(°C) = V_0/10\,mV. \quad (8)$$

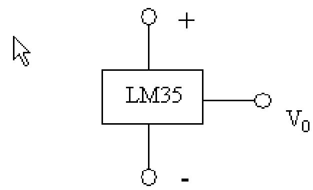


Figure 4 Schematic of temperature sensor LM35.

2.6 Temperature sensor of current-mode (AD590)

AD590 is a current-mode integrated circuit temperature sensor. Its output current is proportional to the temperature with a very good linearity. AD590 temperature sensor has a temperature measurement range of -55 - 150 °C and sensitivity of 1 *μA/K*. It has the characteristics of high accuracy, large dynamic resistance, fast response speed, good linearity and easy to use. AD590 is a two ends device, its circuit symbol is shown in Figure 5.

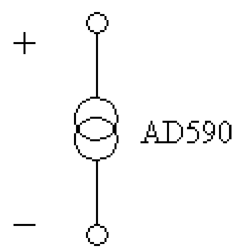


Figure 5 Circuit symbol of AD590

AD590 is equivalent to a high-impedance constant current source, whose output impedance is larger than 10*M* ohm. This can greatly reduce temperature measurement error due to a change of the supplied voltage.

D590 has an operating voltage range of +4 - +30 V and a temperature measurement range of -55 to 150 °C. Corresponding to the thermodynamic temperature *T*, the output current changes 1*μA* for temperature change of 1*K*. Its output current $I_0$ *(μA)* is strictly proportional to the thermodynamic temperature *T (K)*. The current sensitivity is expressed:

$$\frac{I}{T} = \frac{3k}{eR} \ln 8 , \qquad (9)$$

where *k* and *e* are respectively the Boltzmann constant and the electron charge quantity. *R* is the internal integration resistance. Substituting *k/e* = 0.0862 *mV/K*, and *R* = 538 ohm into (9), we get:

$$\frac{I}{T} = 1.000 uA/K . \qquad (10)$$

Therefore, the output current $I_0$ of AD590 represents the thermodynamic temperature value (*K*) of the measured temperature. The output is 273.15 *μA* at temperature *T* = 0 (*K*). The current-temperature (*I-T*) characteristic curve of AD590 is shown in Figure 6:

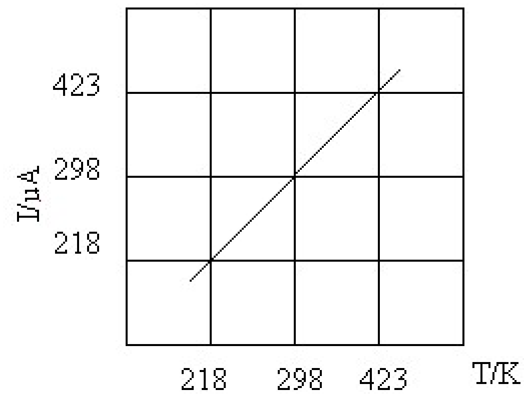

Figure 6 Current-temperature (*I-T*) characteristic curve of AD590

Its output current expression is:

$$I = AT+B, \quad (11)$$

where $A$ is sensitivity and $B$ is the output current at 0 $K$.

If using Celsius temperature t °C, it can be transferred using following formula:

$$T = t\text{-}273.15. \quad (12)$$

In a simple application, taking the benefit of the above mentioned characteristics of AD590, temperature measurement can be fulfilled by using a power supply, a resistor, and a digital voltmeter. The schematic circuit for experimental measurement is shown in Figure 7.

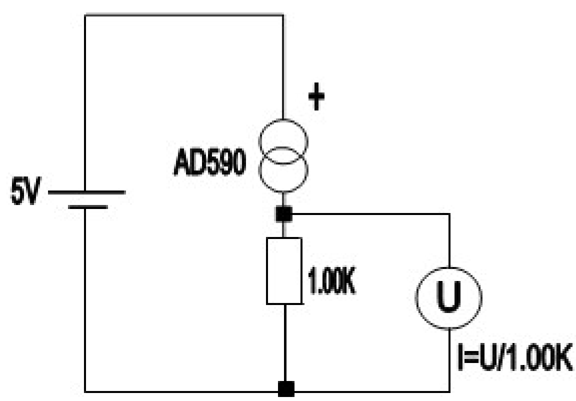

Figure 7 Circuit schematic of AD590 for experimental measurement

2.7 Temperature sensor of PN junction

A PN-junction temperature sensor measures temperature utilizing the characteristics of temperature dependency of the junction voltage of a PN junction. Experiments have proven that under a certain current flowing through, there is a good linear relationship between the forward voltage of the PN junction and the temperature. Usually, by making short circuit between terminals *b* and *c* of a transistor, the PN junction between terminals *b* and *e* is used as a temperature sensor for temperature measurement. The forward voltage $V_{be}$ between terminals *b* and *e* of a silicon transistor is typically approximate 600 mV ( at 25 °C), and is inversely proportional to the temperature with good linearity of temperature coefficient about -2.3 mV/°C.

Normally, the relationship between current $I$ and voltage $U$ of a diode PN junction satisfies the following expression:

$$I = I_S\left[e^{qU/kT} - 1\right]. \tag{13}$$

Under conditions of room temperature and $e^{qU/KT}$ far larger than 1, (13) can be approximated as:

$$I = I_S e^{qU/kT}. \tag{14}$$

In (13) and (14), $q = 1.602\times10^{-19}C$, is the charge of electron, $k = 1.381\times10^{-23}$J/K, is the Boltzmann constant, $T$ is thermodynamic temperature, and $I_s$ is the reverse saturation current.

Under the condition of applying a constant forward current, the relationship between forward voltage $U$ and temperature $t$ is approximately linear as follow:

$$U = Kt + U_{go}, \tag{15}$$

where $U_{go}$ is the semiconductor material parameter and $K$ is the junction voltage temperature coefficient of the PN junction. A schematic of experimental measurement of temperature using a PN junction is shown in Figure 8.

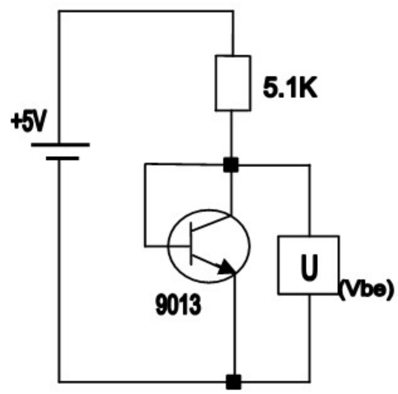


Figure 8 Schematic of experimental measurement of temperature using PN junction