3. Experimental Principle and Contents

3.1 PID applications

3.1.1 Principle and features of PID control

In engineering, the most widely used control approach is proportional, integral and differential control, known as PID control. With features of simple structure, stable, reliable, and easy to adjust, it becomes one of the main techniques in industrial control. PID control also includes PD and PI controls in practice. PID control is based on system error between set and actual values to calculate control quantities using proportional, integral and differential operations.

Proportional (P) control: proportional control is the easiest way of control. The output of the controller is proportional to the difference between set and actual values. However, when only proportional control is used, there is a steady-state error in the system output.

Integral (I) control: to eliminate the steady-state error when only proportional control is used, an "integral term" must be introduced into the system. This integral term increases with time. Under such case, even if the error is very small, the integral term becomes a large value with time to counteract the steady-state error until it is close to zero. Therefore, proportional plus integral (PI) controller allows the system to be free from steady-state error.

Differential (D) control: in the process of an automatic control system to overcome errors, oscillation or instability may occur due to system inertia or delay falling behind the change of errors. A solution to this problem is to inhibit the error in "advance", i.e. when error closes to zero, the impact for overcoming error should become zero. Hence, it is not only to introduce a "proportional term", but also to introduce a "differential term" in the controller to predict the trend of error changes. This way, proportional plus differential in a controller can make the error control to zero or negative in advance, thus avoiding over control. So proportional plus differential (PD) controller can improve system dynamics in the adjustment process.

3.1.2   Parameter tuning of PID controller

Parameter tuning of a PID controller is the core content in design of a control system. It is to determine the quantities of proportional coefficient, integral time and differential time of the PID controller based on the characteristics of the control process. Approaches for parameter tuning of a PID controller can be summarized into two categories: one is the theoretical calculation method based on the mathematical model of the system to determine the controller parameters. Parameters derived from the theoretical method may not be directly applied without modification in practice. The second is the engineering tuning method, which relies mainly on engineering experience, directly conducted in experiments of the control system.

Using engineering tuning method to acquire PID controller parameters includes critical ratio, response curve and decay. Generally, the critical ratio method is usually used. The steps are as follows: (1) select a sampling period that is short enough to let the system work; (2) only add proportional coefficients, until the system displays critical oscillation to a step signal input, at this time, write down the proportional coefficient and the critical period; (3) under a certain control level, use formula to calculate the control parameters of the PID controller.

3.1.3   Applications of PID controller

Currently, in physics experiments and industrial production, stable pressure, temperature, flow, liquid level and speed are usually required. PID controllers are widely used in these areas.

3.1.4 Temperature control of PID controller

PID temperature controller is a widely used for temperature control. P represents proportional coefficient. When temperature has regular fluctuations (oscillations), P should be increased; when temperature makes irregular drift, P should be reduced. I represents integral time. When temperature has regular fluctuations (oscillations), I should be increased; when the steady-state error cannot be eliminated over time, I should be reduced. D represents differential time. Increasing D is beneficial to reducing temperature over the set value. Thus, a PID controller consists of a proportional element (P), an integral element (I) and a differential element (D).

Its input $e(t)$ and output $u(t)$ have relationship:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

where Kp is the proportional coefficient, and Ki is the integral time constant, and K_d is the differential time constant.

3.2 Measurement of heat conductive coefficient of poor conductor

3.2.1 Definition of heat conductive coefficient

Based on the heat conduction theory proposed by Fourier in 1882, if a temperature gradient exists in an object, heat will transfer from high temperature region to lower temperature region. Further, heat amount ΔQ flowing through section area S within time Δt is proportional to the temperature gradient inside the object as

$$\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = -\lambda \frac{dT}{dz} S \tag{1}$$

where dQ/dt is the heat transfer rate, dT/dZ is the temperature gradient in the direction perpendicular to area S, λ is the heat conductive coefficient in unit of W/(m·K), and the minus sign means heat transfers from high temperature region to low temperature region.

3.2.2 Conductive coefficient measurement of poor conductor

As shown in Figure 3, sample plate P and heat dispensing plate B have the same dimensions, i.e. thickness h, cross-section area S ($S=\pi D^2/4$), and diameter D. The temperatures at the upper and lower surfaces of sample plate P are stabilized as T_1 and T_2, respectively. From (1), heat transfer rate can be written as:

$$\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = -\lambda \frac{T_2 - T_1}{h} S = \lambda \frac{T_1 - T_2}{h} S \tag{2}$$

To reduce the effect of heat dispensing on side surface, thickness h of sample plate P cannot be too large. Since temperatures T_1 and T_2 at the upper and lower surfaces of plate P are represented by the temperatures at the bottom of heating plate A and the top of heat dispensing plate B, good thermal contact must be maintained among these plates.
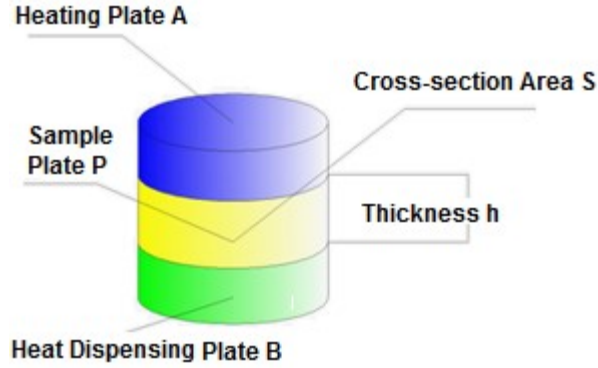
Figure 3 Schematic of heat conductive coefficient measurement system

The method used to measure the heat conductive coefficient of sample plate P is the so-called steady-state method by achieving a stable temperature distribution between any two plates in contact. As a result, temperatures T_1 and T_2 are stable so that the heat transfer rate through sample plate P is equal to the heat dispensing rate (cooling rate) through heat dispensing plate B. Under such case, we can get the heat dispensing rate of heat dispensing plate B instead. After stable T_1 and T_2 are recorded, one can remove sample plate P while letting heating plate A and heat dispensing plate B contact directly. After the temperature of plate B rises by 10 °C, remove heating plate A to let heat dispensing plate B cool down in air while recording the temperature of plate B over time to find the heat dispensing rate of plate B around temperature T_2. Since heat dispensing rate is proportional to the area of all dispensing surfaces exposed to air, i.e. the upper, lower and side surfaces.

In a steady-state, the upper surface of heat dispensing plate B is covered by sample plate P. So, the heat dispensing rate needs to be corrected as follows:

$$\frac{dQ}{dt} = \frac{\frac{\pi D^2}{4} + \pi Dh}{\frac{\pi D^2}{2} + \pi Dh} \cdot \frac{dQ_{all}}{dt} \tag{3}$$

where dQ_{all}/dt is the heat dispensing rate (cooling) in air with all surfaces) as:

$$\frac{dQ_{all}}{dt} = -Cm\frac{dT}{dt} \tag{4}$$

where C, m are the specific heat capacity and mass of the heat dispensing plate, respectively. From (2), (3) and (4), the heat conductive coefficient can be derived as:

$$\lambda = -Cmh\frac{\frac{D}{4} + h}{\left(\frac{D}{2} + h\right)(T_1 - T_2)} \cdot \frac{4}{\pi D^2} \cdot \frac{dT}{dt} \tag{5}$$

where parameters m, D, h, T_1 and T_2 can be measured in experiment. It can be seen from (5) that once cooling rate dT/dt at steady-state is acquired, heat conductive coefficient λ can be calculated.

3.2.3 Measurement of heat conductive coefficient of poor conductor

a. Wiring:

Connect the "T-Control" socket on the back of the main controller to the socket on back of the heating plate on the Module with the 5-pin/5-pin cable.

Connect the "DC 5V" output on the back of the main controller to the back of the Module base (for fan) with the pair of red/black wires.

Connect one temperature sensor to any one sensor input socket of the main controller. Insert the sensor probe into the hole of the heating plate using thermal grease to ensure good thermal contact. Set the corresponding display meter to "Temperature" function.

Connect the other temperature sensor to another input socket of the main controller. Insert the sensor probe into the hole of the heat dispensing plate (aluminum plate) using thermal grease to ensure good thermal contact. Set the corresponding display meter to "Temperature".

Connect electricity power supply to the main controller.

b. Lift and keep the heating plate at the top of the stand posts. Place the heat dispensing plate onto the stand support. Don't place the rubber plate yet. Turn on power of the control unit. Push once on the PID button, and set the heating temperature (e.g. 80 °C), now the heating plate starts heating. Turn on the fan of the experimental module #1. Now, the temperatures of heating plate and heat dispensing plate are displayed on the corresponding meters on the control unit.

c. When the temperature of the heating plate reaches the set temperature, place the rubber plate sample onto the heat dispensing plate. Lower the heating plate. Make sure the three plates are aligned entirely (Note: the holes for mounting sensors on the heating plate and the heat dispensing plate should be as close as possible). Turn the two fixing nuts to achieve good contact between these plates, but avoid overtightening.

d. Record temperature over time (in every minute for 10 minutes or longer) until the temperatures of heating plate and heat dispensing plate are stabilized to reach a steady state. At this time, record temperature of heat dispensing plate T_2 and temperature of heating plate T_1.

e. Press PID button once again to stop heating (make sure at this time the "Manual" knob is turned in the end of count clockwise direction to turn off manual heating). Remove sample plate. Turn off the fan. Turn fixing nuts to make good contact between the heating plate and heat dispensing plate. Remain the PID/Manual button at "Manual" position, the temperature of heat dispensing plate will rise by absorbing the heat of the heating plate, or in case of temperature rising too slow, turn the "Manual" knob in clockwise direction to heat the heat dispensing plate (heating speed can be adjusted by the tuning amount of the knob), until the temperature of the heat dispensing plate is 10°C over the steady-state value T_2.

f. Remain "Manual" position of the PID/Manual button (also the "Manual" knob at minimum position). Lift the heating plate. Turn on the fan. Let the heat dispensing plate cool in air with fan running while recording 10 temperature values of the heat dispensing plate with an interval of 30 seconds. According to the recorded data, plot

cooling curve. Draw a tangent line of the curve at T_2 point. Find the slope of the tangent line and calculate the heat dispensing rate.

g. According to the measured temperature values T_1 and T_2 at steady-state, as well as the heat dispensing rate at temperature T_2, use equation (5) to calculate the heat conductive coefficient of the poor conductor sample.

3.2.4 Measurement of heat conductive coefficient of air

a. Wiring: same as the previous experiment.

b. Remove the rubber sample. Lift the heating plate. Turn on power of the control unit. Press PID button once and turn on the fan, indicator lamp on and the heating plate starts heating. Now, the temperatures of heating plate and heat dispensing plate are displayed on the corresponding meters on the control unit.

c. When the temperature of the heating plate reaches the set temperature (such as 80 ℃), lower the heating plate and place the 1 mm thickness standard plate between the heating plate and the heat dispensing plate. Turn to adjust the height of the three support nuts to touch with the underneath surface of the skirt of the heating plate assembly. Remove the standard plate to create an air gap of 1 mm between the two plates.

d. Repeat experimental steps d, e, f and g as described in previous experiment.

e. Calculate the heat conductive coefficient of air.

3.3 Measurement of heat conductive coefficient of metal

3.3.1 Heat conductive coefficient measurement of metal

If a metal cylinder has high temperature at one end and low temperature at the other end. Heat flows from the high temperature end to the low temperature end. If the high temperature end is heated over time while the temperature does not change within the cylinder, and the dispensed heat from its side surface is negligible, and the heat flowing through each cross-section of the cylinder should be same. This state is called the stable flow of heat.

As shown in Figure 4, if the heat passing through cross-section A_1B_1 is more than that passing through cross-section A_2B_2, heat would accumulate between the two cross-sections leading to a rise in temperature. This means if temperature remains unchanged throughout the cylinder, the heat passing through all cross-sections must be same.
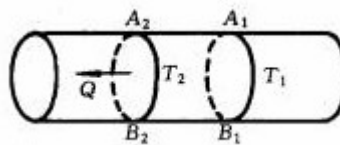

Figure 4 Schematic of measuring heat conductive coefficient of metal

Under steady flow condition, from Fourier heat transfer equation, the heat power flowing through every cross-section of a cylinder is:

$$\frac{dQ}{dt} = \lambda S \frac{T_1 - T_2}{l} \tag{6}$$

where S is the cross-section area of the cylinder, T_1 and T_2 are temperature values at cross-sections A_1B_1 and A_2B_2, respectively, l is the distance between the two cross-sections, factor λ is the heat conductive coefficient of the of the metal under test.

The value of heat conductive coefficient λ represents the heat amount flowing through a unit cross-section area within a unit time in a unit length with a temperature difference of 1°C. It is apparent from (6) that the key to measure the heat conductive coefficient of a good conductor is to maintain a stable flow and measure the heat power passing through the cylinder.

In experiment, when steady-state is reached, the upper surface of the heat dispensing plate is covered by the sample. So the heat dispensing rate should be modified as follows:

$$\frac{dQ}{dt} = \frac{\frac{\pi D^2}{2} + \pi Dh - \pi d^2}{\frac{\pi D^2}{2} + \pi Dh} \cdot \frac{dQ_{all}}{dt} \tag{7}$$

where d is the diameter of the cylinder.

Similarly, by substituting (7) into (4), we get:

$$\lambda = -Cm \cdot \frac{2D^2 + 4Dh - d^2}{2D^2 + 4Dh} \cdot \frac{4l}{\pi d^2 (T_1 - T_2)} \cdot \frac{dT}{dt} \tag{8}$$

where T_1 and T_2 can be accurately measured using digital temperature sensors.

3.3.2   Experiment contents of heat conductive coefficient measurement of metal

Note: the aluminum cylinder sample comes with a plastic/aluminum hollow shell. Place the aluminum cylinder sample in the center of the hollow shell and align their side holes for sensor installation.

In this experiment, the heat capacity of an aluminum cylinder sample is small with limited heat dispensing so a relatively low temperature (e.g. 65 °C) can be set in the experiment. However, this experiment does require stable ambient temperature with minimal air convection without fan or AC cooling. To ensure evenly heating of the aluminum cylinder sample, thermal grease should be applied evenly on both ends of the aluminum cylinder.

Unlike a poor conductor, the temperature change of the aluminum cylinder sample is very fast, so it is difficult to record the temperature difference at steady-state between the two ends of the cylinder. In practice, as long as the temperature difference between the two ends of the cylinder is smaller, it is considered the aluminum cylinder has reached a steady-state. Experiment steps are as follows:

a. Wiring: same as the previous experiment but the two temperature sensor probes should be inserted to the holes at the two ends of the aluminum cylinder, respectively (thermal grease should be applied to the sensor probes to ensure good thermal contact).

b. Lift the heating plate to the top of the stand posts. Don't place the aluminum cylinder yet. Turn on power of the control unit and record the temperature difference between the two sensors in the unheated sample as ΔT_0. Press PID button once and turn on the fan of module # 1, indicator lamp on, the heating plate starts heating.

c. When the temperature of the heating plate reaches the set temperature (such as 65 °C), place the aluminum cylinder sample with shell between the heating plate and the heat dispensing plate following these steps: 1) align the outside edge of the centering plate to the outside edge of the heat dispensing plate; 2) place the aluminum cylinder with shell on the heat dispensing plate, and let the shell in contact with the inner edge of the centering plate; and 3) remove the centering plate. Then lower the heating plate onto the sample cylinder.
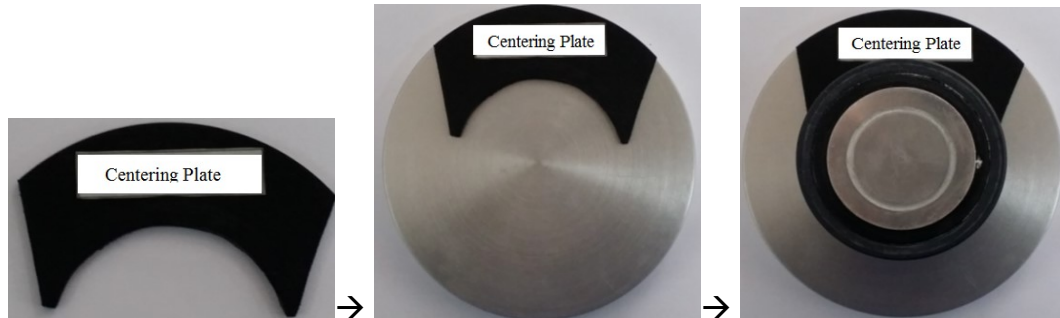

Figure 5 Schematic of centering aluminum cylinder on heat dispensing plate

d. Record temperature difference every 15 seconds until the temperature difference gets small (from large). Now, record temperature T_2 of the heat dispensing plate. Note: the actual temperature difference equals the measured value minus the initial ΔT_0.

e. Press PID button once again to stop heating (make sure at this time the "Manual" knob is turned in the end of count clockwise direction to turn off manual heating). Remove sample cylinder. Turn off the fan. Turn fixing nuts to make good contact between the heating plate and the heat dispensing plate. Remain the PID/Manual button at "Manual" position, the temperature of heat dispensing plate will rise by absorbing the heat of the heating plate, or in case of temperature rising too slow, turn the "Manual" knob in clockwise direction to heat the heat dispensing plate (heating speed can be adjusted by the tuning amount of the knob), until the temperature of the heat dispensing plate is 10°C over the steady-state value T_2.

f. Remain "Manual" position of the PID/Manual button (also the "Manual" knob to minimum position). Lift the heating plate. Turn on the fan. Let the heat dispensing plate cool down in air with fan running. Record 20 temperature values of the heat dispensing plate with time interval 30 seconds each and plot the cooling curve. Draw a tangent line of the curve at T_2 point. Find the slope of the tangent line and calculate the heat dispensing rate.

g. From measured temperature T_1 and T_2 at the steady-state, and the heat dispensing rate at temperature T_2, the heat conductive coefficient of the metal sample can be derived from (8).

3.4 Measurement of specific heat capacity of metal

3.4.1 Specific heat capacity measurement of metal

For a material with a unit mass, the required heat for temperature to increase or decrease by 1 K/°C is called the specific heat capacity of the substance. Its value changes with temperature. According to Newton's law, using cooling method to measure metal specific heat capacity is

one of the most common methods. If the specific heat capacities at different temperatures of a standard sample are known, the specific heat capacity of a sample under test can be measured from its cooling curve.

This experiment uses copper as the standard sample to measure the specific heat capacity of aluminum at 80 °C or 100 °C. After heating a metal sample of mass M_1, placing it into a medium of lower temperature such as air at room temperature, the sample will gradually cool down. Its heat loss within unit time (dQ/dt) is proportional to the rate of temperature decline, so we get the following relationship:

$$\frac{dQ}{dt} = C_1 M_1 \frac{d\theta_1}{dt} \tag{9}$$

where C_1 and dθ_1/dt are the specific heat capacity and the rate of temperature decline of the metal sample at temperature θ_1, respectively. According to the cooling law, we have:

$$\frac{dQ}{dt} = a_1 s_1 (\theta_1 - \theta_0)^m \tag{10}$$

where a_1 is the heat exchange coefficient, S_1 is the area of outside surface of the sample, m is a constant, θ_1 is the temperature of the metal sample, and θ_0 is the temperature of surrounding medium. From equations (9) and (10), we get:

$$C_1 M_1 \frac{d\theta_1}{dt} = a_1 s_1 (\theta_1 - \theta_0)^m \tag{11}$$

Similarly, for another metal sample of mass M_2 and specific heat capacity C_2, we get:

$$C_2 M_2 \frac{d\theta_2}{dt} = a_2 s_2 (\theta_2 - \theta_0)^m \tag{12}$$

From (11) and (12), we have:

$$\frac{C_2 M_2 \dfrac{d\theta_2}{dt}}{C_1 M_1 \dfrac{d\theta 1}{dt}} = \frac{a_2 s_2 (\theta_2 - \theta_0)^m}{a_1 s_1 (\theta_1 - \theta_0)^m} \tag{13}$$

Therefore:

$$C_2 = C_1 \frac{M_1 \frac{d\theta_1}{dt_1} a_2 s_2 (\theta_2 - \theta_0)^m}{M_2 \dfrac{d\theta_2}{dt_2} a_1 s_1 (\theta_1 - \theta_0)^m} \tag{14}$$

If the two samples have the same shape and size, i.e. $s_1 = s_2$, with the same surface conditions (such as coating, color, etc), in the same ambient medium (air), then $\alpha_1 = \alpha_2$, θ_0 is a constant, and $\theta_1 - \theta_2 = \theta$, the above formula can be simplified as follows:

$$C_2 = C_1 \frac{M_1 \left(\dfrac{d\theta}{dt} \right)_2}{M_2 \left(\dfrac{d\theta}{dt} \right)_1} \tag{15}$$

Therefore, if the specific heat capacity C_1 and mass M_1 of the standard sample are known, the specific heat capacity C_2 of the sample under test of mass M_2 can be acquired by measuring the cooling rates (rate of temperature decline) at temperature θ.

3.4.2   Experiment contents of specific heat capacity measurement of metal

a) Wiring: same as experiment 3.2.

b) Place the copper plate onto the stand. Lower the heating plate onto the heat dispensing plate (copper plate), turn the fixing nuts to make good contact between the heating plate and the heat dispensing plate. Apply thermal grease to a sensor tip and fully insert the sensor tip into the hole of the heat dispensing plate. Press PID button once to start heating, the indicator lamp is on. Set the control temperature of the PID around 120 °C, but stop heating when the temperature of the heat dispensing plate reaches 90 °C.

c) Press PID button once again to stop heating, and indicator lamp is off. Release fixing nuts and lift the heating plate. Let the heat dispensing plate cool in air. Use the timer of the control unit, select timing scale 0.01, and measure time t that it takes for the temperature to decline from 84 °C to 80 °C.

d) Replace the heating dispensing plate (cooper) with the aluminum plate sample, repeat Steps a) to c).

e) Repeat 5 times of the above steps for both copper and aluminum samples. Since the temperature decline range is the same ($\Delta\theta = d\theta = 84°C-80°C = 4°C$), Equation (15) can be simplified as:

$$C_2 = C_1 \frac{M_1 t_2}{M_2 t_1} \tag{16}$$

As the specific heat capacity of copper C_1 is known, C_2 of aluminum can be determined.

3.5 Measurement of linear expansion coefficient of metal

3.5.1   Linear expansion coefficient measurement of metal

The phenomenon of length increase of a solid object when heated is called linear expansion of solid. Experiments show that a solid of length L after being heated, its relative elongation dL/L is proportional to temperature change dt, as:

$$\frac{dL}{L} = \alpha dt \tag{17}$$

where α is called the linear expansion coefficient of the solid which is an inherent property of the material. The linear expansion coefficients of several materials are given in the table below.

Material	Copper, iron, aluminum	Glass, ceramic	Invar steel	Fused silica
α	$\approx 10^{-5}\,(°C)^{-1}$	$\approx 10^{-6}\,(°C)^{-1}$	$< 2 \times 10^{-6}\,(°C)^{-1}$	$\approx 10^{-7}\,(°C)^{-1}$

From (17), after measuring length L at T_1, and elongation ΔL when temperature changes from T_1 to T_2, the linear expansion coefficient of the material in temperature region (T_1, T_2) is:

$$\alpha = \frac{\Delta L}{L(T_2 - T_1)} \tag{19}$$

The key for measurement of linear expansion coefficient is to measure the elongation amount ΔL which is in the sub-millimeter range. For such a tiny of elongation, micrometer, reading microscope, optical level, or optical interference is used. This experiment uses a micrometer.

3.5.2 Experiment contents of linear expansion coefficient measurement of metal

a. Wiring: same as experiment 3.2.

b. Place the copper plate onto the stand. Lower the heating plate. Turn the fixing nuts to make the heating plate and the heat dispensing plate (copper plate) in good contact. Apply thermal grease to the tip of the sensor and fully insert the sensor tip into the hole of the heat dispensing plate. Fix the micrometer to the micrometer mounting hole using the micrometer fixing screw. Adjust the positioning screw (on the other side) to push the heat dispensing plate a little forward to let the micrometer tip attach to the heat dispensing plate and the micrometer has a certain initial preset reading. Record the temperature of the heat dispensing plate as T_1 and micrometer reading as l_1.

c. Press PID button once to start heating and set the PID control temperature at 120 °C.

d. Record temperature and the corresponding expansion amount such as 40 °C, 45 °C, 50 °C, 55 °C, 60 °C, 65 °C, 70 °C, 75 °C, ... and the corresponding micrometer readings L40, L45, L50, L55, L60, L65, L70, L75, ... Generally record 10 set data. Then calculate elongation values ΔL and temperature differences ΔT.

e. Use formula $\alpha = \dfrac{\Delta L}{L\Delta T}$ to calculate the expansion coefficient of the copper plate (**note**: L is the diameter of the copper plate).

f. Replacement the copper plate with the aluminum plate, repeat the above steps to acquire the heat expansion coefficient of aluminum.