## 2. Theory

For one mole of an ideal gas, the relation between the specific heat in constant pressure $C_p$ and the specific heat in constant volume $C_v$ is as follows:

$$C_p - C_v = R \tag{1}$$

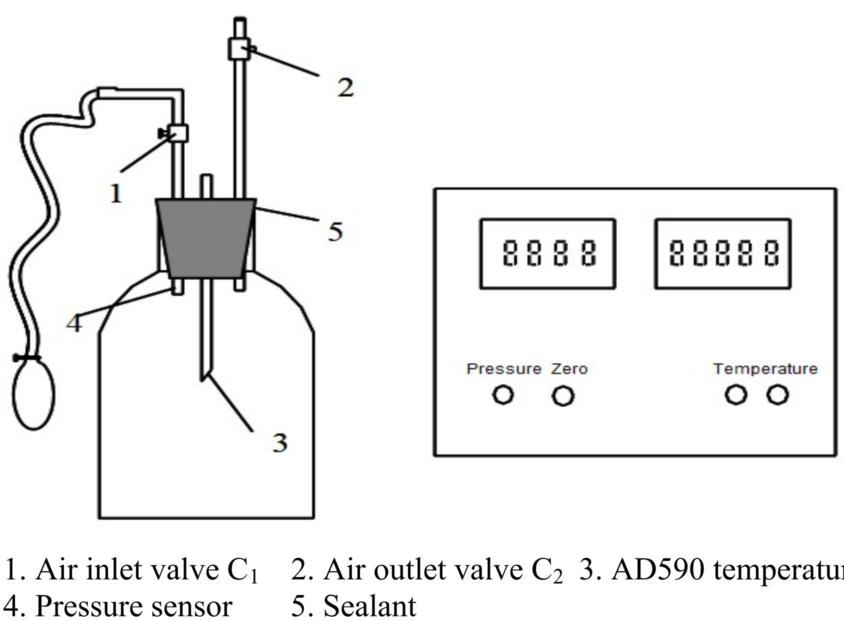
where $R$ is the universal gas constant. The specific heat (capacity) ratio $\gamma$ of a gas is:

$$\gamma = \frac{C_p}{C_v} \tag{2}$$

The specific heat ratio of gas $\gamma$ is also called the adiabatic coefficient of gas. It is an important physical quantity, and often appears in various thermodynamic equations.

A schematic of the apparatus for measuring the value of $\gamma$ is shown in Figure 1, whereas the experimental process is shown in Figure 2.

By adopting the remaining air in the container bottle at State II as the object under study, the experimental process is carried out as following (where $P_0$ is the ambient atmospheric pressure, $T_0$ is the room temperature, and $V_2$ is the volume of the bottle container):




1. Air inlet valve $C_1$ 2. Air outlet valve $C_2$ 3. AD590 temperature sensor
4. Pressure sensor 5. Sealant

Figure 1 Schematic of apparatus for measuring specific heat ratio of air

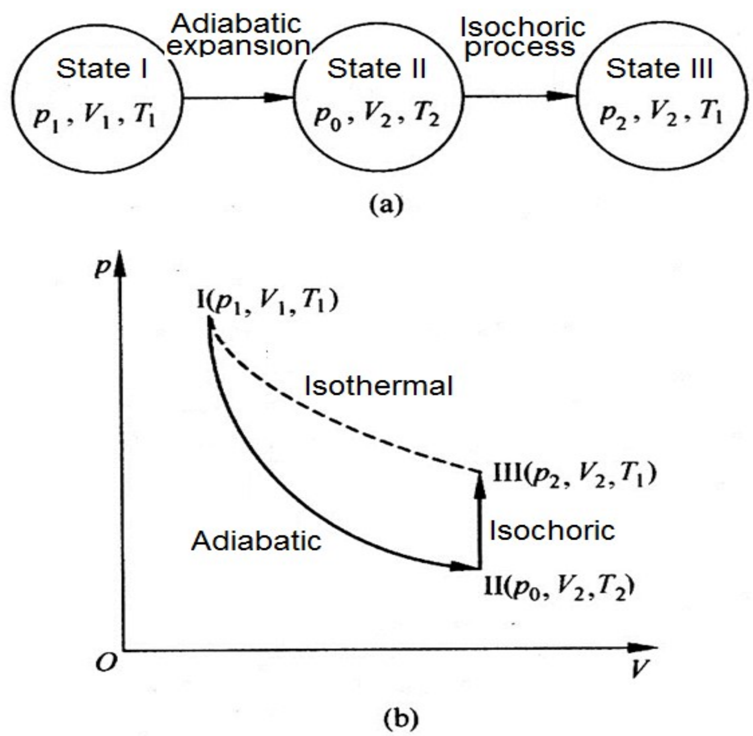



Figure 2 Process flow-chart for measuring specific heat ratio of air

1) Open valve $C_2$ to let air into the bottle container. Now, the air in the bottle has the same temperature and pressure as the ambient air. Then close $C_2$.

2) Open inlet valve $C_1$, pump some air into the bottle with the inflating ball and then close inlet valve $C_1$. Now, the air in the bottle is compressed with both pressure and temperature increased. Then, the air temperature will slowly drop as heat exchanges with the pumped air. Wait for the air temperature in the bottle to be stabilized at $T_1$, which should reach $T_0$ if the waiting time is long enough and the ambient temperature is stable at $T_0$. In practice, however, when the temperature change tends to stop, we can assume $T_1 \cong T_0$, and the resulting error on the experimental results is negligible. Now the gas is in State I ($P_1$, $V_1$, $T_1$). **Note**: the object under study is limited to the remaining air in the container bottle at State II, where volume $V_1$ will be smaller than $V_2$ since there is extra pumped-in air in the bottle apart from the object under study.

3) Open outlet valve $C_2$, let the air bottle open to the ambient air. When the pressure drops to $P_0$, immediately close $C_2$. Since the process of air releasing is very fast, the air in the bottle has almost no heat exchange with the outside air, so it can be approximated as an adiabatic expansion process. Now, the air state in the bottle is transformed from State I ($P_1, V_1, T_1$) into State II ($P_0, V_2, T_2$).

4) After the adiabatic expansion process, air temperature $T_2$ in the bottle is lower than the ambient temperature, so the air in the bottle will slowly absorb heat from the outside air. Wait for the air temperature in the bottle to be stabilized at $T_3$, while recording the air pressure and temperature in the bottle as $P_2$ and $T_3$, respectively. Now, the air state becomes State III ($P_2, V_2, T_3$). The process from State II to State III can be regarded as an isochoric process. Again, $T_3$ should reach $T_0$ if the waiting time is long enough and the ambient temperature is stabilized at $T_0$. In practice, however, when the temperature change tends to stop, we can assume $T_3 \cong T_0$, and the resulting error on the experimental results is negligible. Now $T_0=T_1=T_3$, so State III can be written as ($P_2, V_2, T_1$).

From State I to State II is an adiabatic process, so we have:

$$P_1(V_1)^{\gamma}=P_0(V_2)^{\gamma} \tag{3}$$

State I and State III have the same temperature $T_1$, so we have:

$$P_1V_1 = P_2V_2 \tag{4}$$

By taking logarithmic operation of (3) and then substituting the volume ratio from (4), we get:

$$\gamma = \frac{\ln P_1 - \ln P_0}{\ln P_1 - \ln P_2} = \frac{\ln(P_1/P_0)}{\ln(P_1/P_2)} \tag{5}$$

It is apparent from Equation (5) that the air specific heat ratio can be derived by measuring $P_0$, $P_1$, and $P_2$. The schematic diagram of temperature measurement is shown in Figure 3 using an AD590 as the temperature sensor.

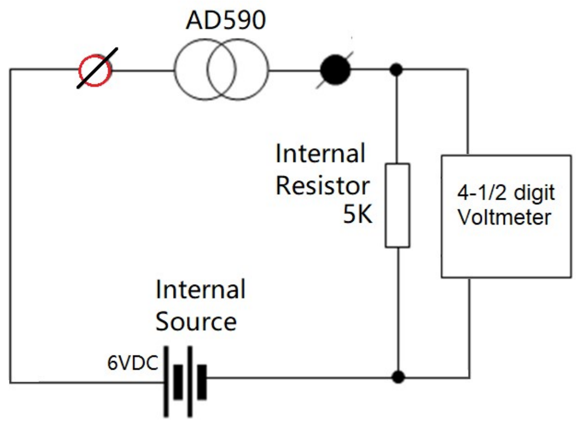



Figure 3 Schematic of temperature measurement using AD590 sensor

A standard 5 kΩ resistor and a 6 VDC voltage source are built in the electric unit. Connecting AD590 temperature sensor with the 6 VDC power supply creates a stable current flow source with a temperature sensitivity of 1 μA/°C; while by connecting the 5 kΩ resistor in series with the AD590 sensor and source, an current-to-voltage converter is achieved with a temperature sensitivity of 5 mV/°C. Using a 4-1/2 digit voltmeter to measure this signal, the temperature sensitivity can be 0.02 °C. If U represents the voltage in unit of mV, the measured temperature in unit of degrees Celsius is:

$$T = \frac{U}{5} - 273 \tag{6}$$