5. Serial Experiments with He-Ne Laser

5.1 Spot size and beam divergent angle measurements of He-Ne laser

Objective:

1. Learn how to measure the spot size and beam divergent angle of a He-Ne laser
2. Understand the Gaussion distribution characteristic of a fundamental mode and the meaning of the divergent angle of a laser beam

Experimental components:

He-Ne laser, optical power meter, photo detector, slit, stand ruler, and micro-displacement stage

Theory of experiment:

Beam divergent angle and cross-sectional beam size are the two important parameters in laser applications. Though a laser beam is highly directional, it is not ideally parallel but divergent over distance. In specific laser applications such as laser collimation and laser range finder, the divergent angle of the laser beam needs to be reduced by adopting a beam expanding telescope.

1. Divergent angle of laser beam θ

The propagation of a laser beam in free space is illustrated in Figure 1 with the location at which the beam spot is the narrowest called the beam waist. As seen in Figure 1, let the origin of cylinder coordinates (z, r, φ) locate at the cross-sectional center of the beam waist while z is the propagating direction of the laser beam.

For a Gaussion beam, the radius of the beam spot over a propagating distance z can be expressed as

$$w(z) = w_0\sqrt{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2} \tag{4}$$

where w_0 is the beam waist, and λ is the laser wavelength. Equation (4) can be rewritten into the equation of a hyperbola, as

$$\left[\frac{w(z)}{w_0}\right]^2 - \left[\frac{\lambda z}{\pi w_0^2}\right]^2 = 1 \tag{5}$$

Equation (5) is plotted as seen in Figure 2. The intersection angle between the two asymptotes of the hyperbola, θ, is defined as the divergent angle of the laser beam. Thus

$$\theta = \frac{2\lambda}{\pi w_0} = \frac{2w(z)}{z} \quad (z \text{ is a large value}) \tag{6}$$

Based on equation (6), if the spot size, $2w(z)$, can be measured over a large distance, z, from the beam waist, then the divergent angle of a laser beam can be calculated.

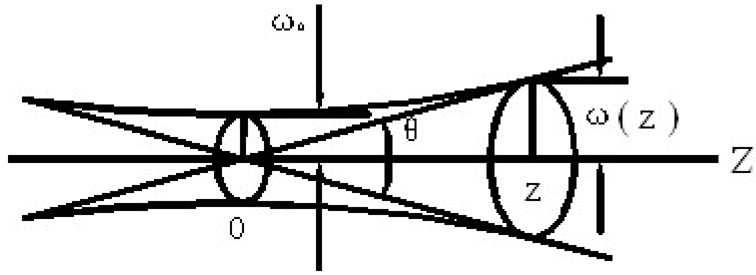

Figure 2 Divergence of a laser beam

2. Transverse optical field distribution of laser beam

For the fundamental mode of a He-Ne laser, the amplitude distribution of its transverse optical field is governed by a Gaussian formula,

$$E_{00}(r) = E_{00}(z)\exp\left[-\frac{r^2}{w^2(z)}\right] \tag{7}$$

Where E_{00}(z) is the amplitude of the optical field at the center in the cross section located with a distance of z from the beam waist, and w(z) is the radius of the beam size located by a distance of z from the beam waist. Expressed by a Gaussian distribution, such laser beam is also called a Gaussian beam. When $r=w$(z), E_{00}(r)=E_{00}(z)/e. Now, beam radius w(z) can be defined as the off-center distance at which the amplitude drops to 1/e of the amplitude at the center.

To get the corresponding intensity distribution, equation (7) is squared to become

$$I_{00}(r) = E_{00}^2(r) = E_{00}^2(z)\exp\left[-\frac{2r^2}{w^2(z)}\right] = I_{00}(z)\exp\left[-\frac{2r^2}{w^2(z)}\right] \tag{8}$$

Alternatively, beam waist w(z) can be defined as the off-center distance at which the intensity drops to $(1/e)^2$ of the intensity at the center

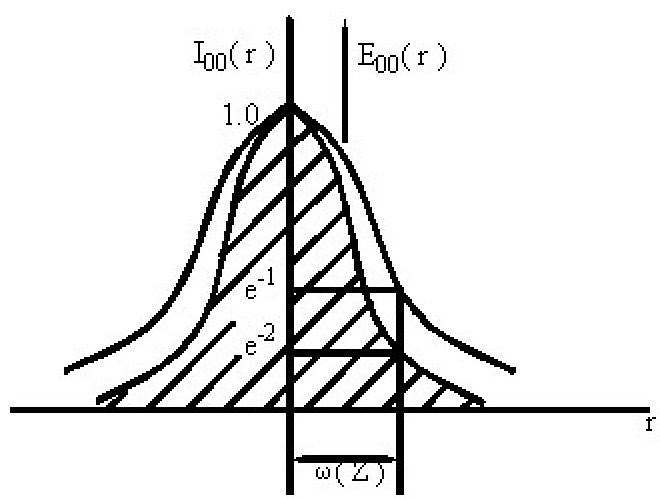

Figure 3 Amplitude and intensity distributions of a Gaussian beam

As seen in Figure 3, roughly 86.5% of total optical power is confined to the spot size with a radius of w(**z**).

3. Beam radius and divergent angle measurements

As seen in Figure 4, the He-Ne laser used in this experiment employs a plane-concave resonant cavity with a cavity length of L and a radius of curvature of R for the concave mirror. The beam waist of such laser can be obtained as

$$w_0 = \sqrt{\frac{L\lambda}{\pi}\sqrt[4]{\frac{R}{L}-1}} \tag{9}$$

Accordingly, the divergent angle of the laser beam can be calculated as

$$\theta = \frac{2\lambda}{\pi w_0} \tag{10}$$

Typically, the beam waist of such laser is located at the position of the plane mirror (output mirror) of the resonant cavity. In this experiment, the beam radius at a distance of 3-5 m from the beam waist is measured as shown in Figure 4. To reduce the size of the experimental setup, folding mirrors are used as seen in Figure 4. A slit followed by a photo detector is moved transversely while photocurrent from the detector is recorded and hence the intensity distribution of the laser can be obtained. From the obtained intensity distribution curve, find the location at which the intensity drops to $(1/\mathbf{e})^2$of the maximum intensity ($\mathbf{e}$=2.72, $\mathbf{e}^{-2}$=0.14) and the beam radius at this location is the beam radius $w(\mathbf{z})$ to be measured. With the beam radius, the divergent angle can be calculated from equation (6).

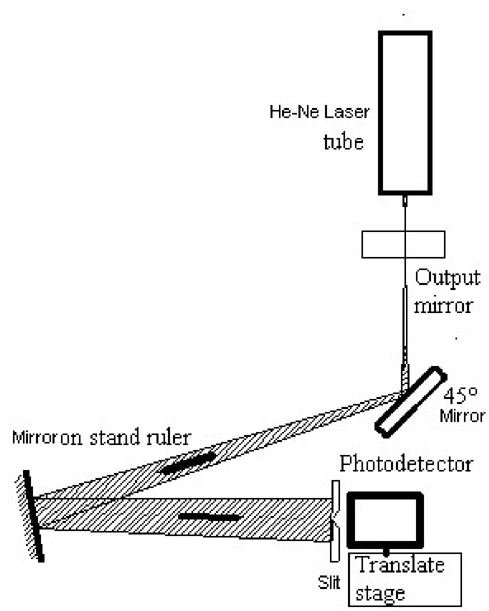

Figure 4 Schematic of experimental setup

Experiment procedure:

1. Preparation

Align optical path following these steps:

a) Mount the alignment He-Ne laser head (the short tube) and the external cavity He-Ne laser head (the long tube with transparent ends) at the two ends of the rail, respectively.

b) Turn on the alignment He-Ne laser. Place the alignment hole onto the rail. Adjust the six adjustable screws of the laser tube holder while moving the alignment hole back and forth along the rail, until the laser beam always points to the center of the hole.

c) Adjust the screws of the external cavity laser tube holder to let the transmitted laser beam from the alignment He-Ne laser go through the alignment hole onto the central area of the front window surface. Adjust the screws of the external cavity laser tube holder while observing the reflected laser spot until the reflected beam hits the center of the alignment hole.

d) Turn on the external cavity laser. Mount the output mirror (R=1 m) onto a 2D carrier and place it in front of the external cavity head on the rail at a proper distance (e.g. a few centimetres). Adjust the output mirror while observing the reflected laser beam from the alignment laser on the surface of the output mirror, until the reflected beam again hits the center of the alignment hole.

e) Now, the external cavity laser should lase. If not, slowly readjust the output mirror, to let the reflected laser spot derive a little from the alignment hole until the external cavity laser start lasing. Finally, finely align the output mirror to maximize the laser power output.

2. Configuration of experiment

Per Figure 4, remove the other output mirror (R=∞) by mounting the 45° mirror instead onto a 2D holder and place it on the rail. Mount the flat mirror onto a 2D holder (SZ-07) with mirror surface facing back, assemble the stand ruler onto the tripod, and mount the SZ-07 onto the holder on the back of the ruler. Mount the photo detector to the translation stage on the rail, and attach the silt to the detector. Adjust the total path length from the output mirror to the slit between 3 and 5 m. Adjust optical path to let the laser beam illuminate on the slit and adjust the slit width at roughly 1/10 of the spot size. Connect the photo detector with the photocurrent amplifier.

3. Measurement of transverse intensity distribution

Use the micro-displacement stage to move the slit and photo detector simultaneously across the light spot while maintaining the stage movement perpendicular to the propagation direction of the laser beam. Laser power reading should be recorded in a step of 0.1 ~ 0.2 mm. Repeat the measurement three times and then use averaged data to plot the intensity distribution curve. In the meantime, use the tape measure to measure the **z** value.

4. Determination of beam radius $w(\mathbf{z})$ and divergent angle θ

Use the obtained intensity distribution curve to determine the beam radius $w(\mathbf{z})$ and then use equation (3) to derive the divergent angle. Also, calculate the divergent angle using equations (9) and (10), and then compare with the results.

Warning: 1. Avoid direct eye exposure to laser beam.
2. Due to high-voltage, do not open the laser power supply when powered on.
3. Minimize vibration of the setup to avoid the shifting of light spot on the slit.

5.2 Modal analysis of He-Ne laser with confocal spherical scanning interferometer

Objective:

1. Understand the principle and operation of a scanning interferometer
2. Learn how to observe and measure the longitudinal and transverse modes of a laser

Experimental components:

Scanning interferometer, high-speed photo-detector, ramp generator, oscilloscope, He-Ne laser

Theory of experiment:

A confocal spherical interferometer was first proposed by Alain Connes in 1958 based on the theory of multi-beam interference, and it has been widely used as the resonant cavity of a variety of lasers. In the meantime, it can be used to analyze the output spectral characteristics of lasers for various applications. For example, holographic photography and laser collimation need the use of single transverse mode lasers; while laser ranging and frequency-locking applications call for not only single transverse mode but also single longitudinal mode lasers. Such interferometer is usually scanned via piezoelectric or pressure-operated means with a resolution exceeding 10^7.

Confocal structure has many attractions. Due to the excellent mode degeneracy within a confocal cavity, strict mode matching is not necessary for a confocal cavity. Moreover, off-axial light incidence poses no adverse effect on the performance of a confocal cavity. Another benefit from using a confocal cavity is that the tilting of cavity mirrors has no tight tolerance. Furthermore, the diffraction loss of a confocal cavity is low with a small spot size on the cavity mirrors, hence reducing the coating requirements of mirrors for mass production and application.

1. Fundamental principle of confocal spherical scanning interferometer

A confocal spherical scanning interferometer consists of two highly reflective spherical mirrors M_1 and M_2 with an identical radius of curvature r. The two mirrors are separated by a distance of L that is equal to the radius of curvature of cavity mirrors so that the foci of the mirrors overlap forming a confocal system. One mirror is attached to a piezoelectric transducer and driven back and forth to move periodically by the transducer when supplied with electric voltage in a sawtooth waveform. So, the length of the cavity is changed continuously to achieve wavelength scan.

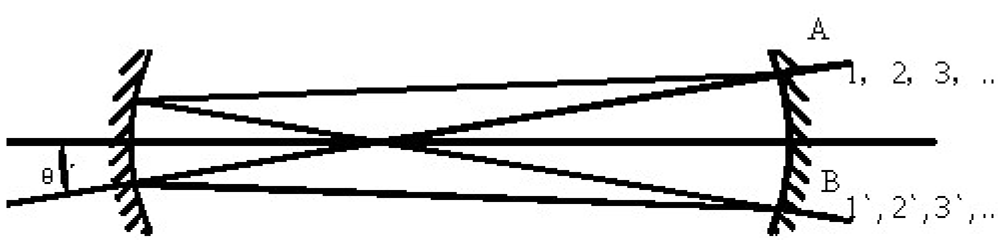

Figure 5 Schematic of confocal cavity

As seen in Figure 5, if a ray of laser light with a wavelength of λ is incident on a confocal cavity at an angle near parallel to the optical axis, the optical ray completes a round trip after reflecting 4 times within the cavity and the length of the round trip is equal approximately to $4L$ (L is the cavity length). Obviously, there are two rays of light transmitted through the cavity, exited from the cavity at points A and B, respectively. The intensities of the transmitted light are

$$I_1 = I_0\left(\frac{T}{1-R^2}\right)^2\left[1+\left(\frac{2R}{1-R^2}\right)^2\sin^2\beta\right]^{-1} \tag{11}$$

$$I_2 = R^2 I_1 \tag{12}$$

where I_0 is the intensity of the incident light, T is the transmissivity of the mirror, and β is the phase delay incurred after one round trip within the cavity expressed as

$$\beta = \frac{4\pi nL}{\lambda} \tag{13}$$

where n is the refractive index of the medium in the cavity. If $\beta=k\pi$ (k is an integer called the order of the scanning interferometer), then

$$4nL = k\lambda \tag{14}$$

Under such condition, a maximum mirror transmittance is achieved as

$$T_{\max} = \frac{I_1}{I_0} = \frac{T^2}{(1-R^2)^2} \tag{15}$$

If the total optical absorbance with the cavity is A, then

$$R + T + A = 1 \tag{16}$$

Substituting equation (16) into (15) by assuming $R\cong 1$, we get

$$T_{\max} \approx \frac{1}{4\left(1+\frac{A}{T}\right)^2} \tag{17}$$

It is apparent from equation (14) that spectral scanning can be achieved by changing either the length of the resonant cavity or the refractive index of the medium in the cavity. The refractive index of a gas medium can be altered by changing the pressure of the gas in the cavity; while the length of the resonant cavity can be scanned by using a piezoelectric transducer attached to one of the cavity mirrors.

2. Oscillation modes of laser

A stable optical oscillation within the resonant cavity of a laser is called a mode, classified into transverse modes and longitudinal modes. A longitudinal mode represents a laser output with a specific frequency while a transverse mode describes the distribution of the optical field along the plane perpendicular to the propagating direction of a laser beam. The spectral linewidth and coherent length of a laser are determined by its longitudinal modes while the beam divergence, spot size, and transverse distribution of energy depend on the transverse modes of the laser.

Typically, symbol "TEM_{mnq}" is used to denote the electromagnetic field formed in the resonant cavity of a laser, where *TEM* represents transverse electromagnetic field, m and n denote the order numbers of a specific transverse mode, and q denotes the order number of a longitudinal mode. Normally, q can be a large quantity while m and n are small integers. The spot pictures of common transverse modes are shown in Figure 6.

TEM_{00} TEM_{10} TEM_{01} TEM_{20} TEM_{11}

Figure 6 Pictures of common transverse modes

(1) Longitudinal modes of laser

Standing waves or stable optical oscillations can be formed within a resonant cavity only when the length of the cavity is a half-integer of the wavelength. Thus

$$L = \frac{q}{2}\lambda \tag{18}$$

where q is the order number of a longitudinal mode. Equation (18) can be rewritten as

$$\nu_q = \frac{qc}{2nL} \tag{19}$$

where ν_q is the frequency of a specific stable oscillation in the laser cavity, and c is the speed of light in vacuum. Based on equation (19), the frequency spacing between adjacent longitudinal modes (Δq=±1) is

$$\Delta\nu = \frac{c}{2nL} \tag{20}$$

Because different longitudinal modes have different gains, only certain longitudinal modes with gains exceeding the threshold can form laser output. For instance, for a He-Ne laser with 1-m cavity, the frequency spacing between adjacent longitudinal modes of such laser is $\Delta\nu=1.5\times10^8$ Hz. If the width of the gain curve of the laser is 1.5×10^9 Hz, a maximum of 10 longitudinal modes can be obtained. The shorter the cavity length the larger $\Delta\nu$ becomes and the fewer longitudinal modes output. With the same gain curve, if the length of laser cavity is shorter than 0.15 m, then only one longitudinal mode can output from the laser. Such laser is called a single longitudinal mode laser.

(2) Transverse modes of laser

For a certain longitudinal mode satisfying the oscillation condition of standing waves, there exist different transverse modes distributing on the planes perpendicular to the propagating direction of the laser beam. The frequencies of different transverse modes corresponding to an identical longitudinal mode are different, as

$$\nu_{mnq} = \frac{c}{2nL}\left\{q + \frac{m+n+1}{\pi}\arccos\sqrt{\left(1-\frac{L}{r_1}\right)\left(1-\frac{L}{r_2}\right)}\right\} \tag{21}$$

where r_1 and r_2 are the radii of curvature of the cavity mirrors, respectively.

The frequency spacing between different transverse modes can be calculated as

$$\Delta\nu_{mn\to m'n'} = \frac{c}{2n\pi L}\left\{(\Delta m + \Delta n)\arccos\sqrt{\left(1-\frac{L}{r_1}\right)\left(1-\frac{L}{r_2}\right)}\right\} \tag{22}$$

By comparing equations (20) and (22), one can come to a conclusion that the frequency spacing between adjacent transverse modes is normally smaller than that between adjacent longitudinal modes. For example, for a laser with a plane-concave (r_1=1 m, $r_2=\infty$) cavity, if the length of the laser cavity is 0.24 m, the frequency spacing between adjacent longitudinal modes is 6.25×10^8 Hz based on equation (20). The frequency spacing between adjacent transverse modes of the fundamental longitudinal mode can be calculated from equation (22) as

$$\Delta\nu_{00\to 01} = \frac{3\times10^8}{2\times1\times3.14\times0.24}\left\{(0+1)\arccos\sqrt{\left(1-\frac{0.24}{1}\right)\left(1-\frac{0.24}{\infty}\right)}\right\} = 1.02\times10^3\ (\mathrm{Hz}) \tag{23}$$

If the width of the gain curve is 1.5×10^9 Hz, then 2.5 longitudinal modes can exist. If a scanning interferometer is used to analyse the mode structure of such laser, three or two peaks will be observed for the TEM_{00} mode; if the TEM_{01} also exists, then three or two peaks, or even a single peak will be observed. These phenomena are caused by the change in the cavity length of the scanning interferometer. So, a scanning interferometer is a powerful tool for laser mode analysis.

3. Characteristics of confocal spherical interferometer

(1). Free spectral range $\Delta\lambda$

By taking derivative operation of equation (14), we get $k\Delta\lambda=-\lambda\Delta k$. if Δk=1, then the free spectral range, $\Delta\lambda$, can be derived as

$$|\Delta\lambda| = \frac{\lambda^2}{4nL} \tag{24}$$

Alternatively, the free spectral range can be expressed in frequency as

$$|\Delta\nu| = \frac{c}{4nL} \tag{25}$$

The meaning of the free spectral range is that the interference fringes do not overlap within a wavelength range from λ to $\lambda+\Delta\lambda$.

(2). Revolving power R_0

The resolving power of an interferometer is defined as the ratio of the wavelength to the spectral linewidth or the minimum wavelength separation, as

$$R_0 = \frac{\lambda}{\Delta\lambda} \tag{26}$$

(3). Finesse F

The finesse of an interferometer is defined as the ratio of the free spectral range to the spectral linewidth, as

$$F = \frac{\Delta\lambda}{\delta\lambda} = \frac{\Delta\nu}{\delta\nu} \tag{27}$$

If both mirrors of a cavity have the same reflectivity R, then equation (27) can be approximated as

$$F = \frac{\pi\sqrt{R}}{1-R} \tag{28}$$

Experiment procedure:

1. Use the concave output mirror (R=1 m) for the external cavity He-N laser. Turn on the laser power, and carefully adjust the output mirror to achieve a maximum laser output. (Note: refer to the experimental preparation section).
2. Place the alignment hole behind the output mirror, followed by scanning interferometer and high-speed photo-detector on the rail.
3. Wire connections: a) connect the scanning interferometer to "Ramp Output" on the back panel of the ramp generator, (b) connect the 4-core cable of the high-speed photo-detector to "Detector" on the back panel of the ramp generator, (c) connect the "Ramp Signal" on the back panel of the ramp generator to one channel of a dual-trace oscilloscope with a BNC cable, (d) connect the BNC cable of the high-speed photo-detector to the other channel of the dual-trace oscilloscope.

   Caution: do not plug/unplug cables when the Ramp Generator is powered on.
4. Adjust the optical path to let the laser beam pass through the alignment hole, adjust the position of the interferometer to let the beam enter the center of the entrance hole (**note**: the front and back caps of the scanning interferometer should be removed). Adjust the tilt of the interferometer mount to make sure the small beam spot reflected from the cavity mirror returns to the center of the aperture hole. This is to ensure the incident He-Ne laser beam coincide with the axis of the scanning interferometer.
5. Align the detector to the output of the scanning interferometer.
6. Turn on the ramp generator and the oscilloscope.
7. Observe the signal on the oscilloscope. Finely tune the screws on the interferometer and the screws on the detector mount to enhance the signal on the oscilloscope.
8. Adjust the amplitude of the ramp voltage while monitoring the change of the interference order on the oscilloscope (The higher the voltage peak value, the more interference order numbers). Keep the peak stable, and confirm the number of interference orders.
9. Properly adjust amplitude, frequency, offset, and rise-time of the ramp generator. According to the number of the modes and the period of the signal, determine how many modes belong to the same interference order of k.
10. Using the definition of free spectral range, confirm the corresponding frequency interval (namely which two spectra have the interval of $\Delta\lambda_{FSR}$?). To reduce measurement error, an increase in the amplitude of the X axis is needed. Measure the length of the scale corresponding to $\Delta\lambda_{FSR}$, and then calculate the frequency interval per centimeter.
11. Observe the longitudinal mode. Compare the definition of longitudinal modes with the characteristic of the spectrum, determine the number of the longitudinal modes, and measure the frequency interval ($\Delta\nu_{\Delta q=1}$) of adjacent longitudinal modes. Compare this to the theory, examine and identify whether the measured value is correct.

12. There are several different transverse modes corresponding to one interference order k. Measure the frequency interval ($\Delta v_{\Delta m+\Delta n}$) of different transverse modes. Compare the experimental results to the theory to check whether the result is correct. Put the results into equation (22) and get the value of $\Delta m+\Delta n$.
13. Make sure the direction of the frequency increase in the horizontal axis, so a high-order transverse mode and a low-order transverse mode corresponding to the same longitudinal mode order will be found. The intensity relation of them can also be derived.
14. View the laser spots on a white screen from distance, the entire transverse modes can be seen. Compare to the shapes of the single transverse modes from Figure 6 to confirm the order value of m and n of every transverse mode.
15. Replace the concave output mirror (R=1 m) with the plane output mirror ($R=\infty$). Repeat the above experiment. Observe two laser cavities with different mode patterns, and then summarize the basic method of mode analysis.

Calculation:

1. Based on the cavity length of the laser, use equation (20) to calculate the frequency spacing of adjacent longitudinal modes of the laser, and then use equation (22) to calculate the frequency spacing between a 1^{st}-order and a 2^{nd}-order transverse mode.
2. Based on the radius of curvature of the mirrors for the scanning interferometer, calculate the free spectral range of the interferometer, and then calculate the finesse of the interferometer with the given reflectivity of the mirrors.
3. Use the calculated free spectral range to calibrate the oscilloscope, and then measure the linewidth of a longitudinal mode on the oscilloscope. Calculate the finesse of the interferometer using equation (27) and compare the calculated finesse with the theoretical value derived from equation (28).