4 THEORY

4.1 Interaction of Light and Matter

The interaction between light and matter is basically recognized as the interaction between light and atoms. There are three processes involved: absorption, spontaneous emission and stimulated emission. In the beginning, an atom remains in its ground state E_1 if not excited by an external photon. When a photon with energy of $h\nu_{21}$ is incident on an atom, the atom may absorb the photon's energy and reach an excited state E_2. In this process, photons with energy equal to E_2 -E_1 (=$h\nu_{21}$) may be absorbed.

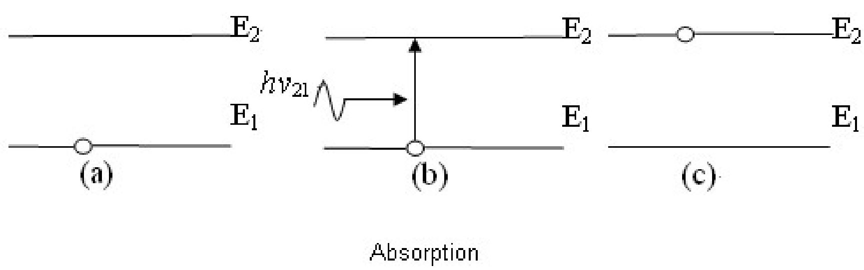

Figure 2 Energy diagram of absorption process

The lifetime of an excited state is so short that the excited atom spontaneously returns to the ground state and emits a photon. Spontaneous emission is not related to external actions and emissions of individual photons are independent, therefore these emitted photons have random phases and polarization orientations.

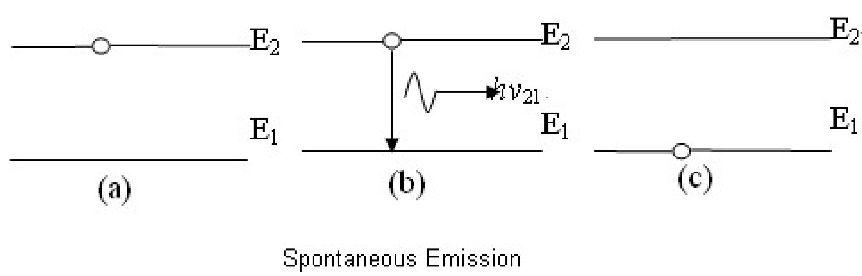

Figure 3 Energy diagram of spontaneous emission

An atom in an excited state will transit from a higher energy state to a lower energy one under external photon stimulation, giving off a photon whose energy equals to the energy difference between the two energy states. This process is called stimulated emission. In this process, the stimulated photons have the same frequency, polarization, and phase as the original photon. Laser generation depends on simulated emission.

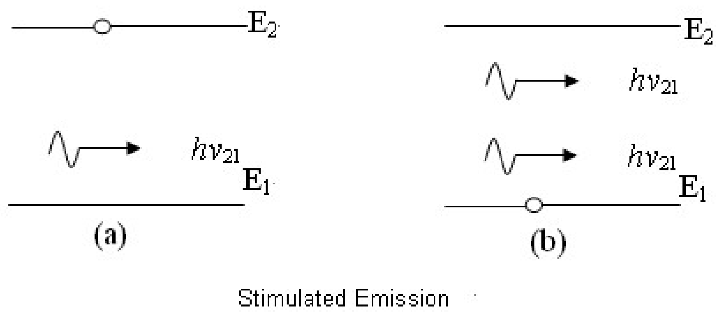

Figure 4 Energy diagram of stimulated emission

4.2 Laser Principle

A laser system consists of three components: a gain medium, a resonant cavity, and a pumping source. The gain medium provides population inversion.

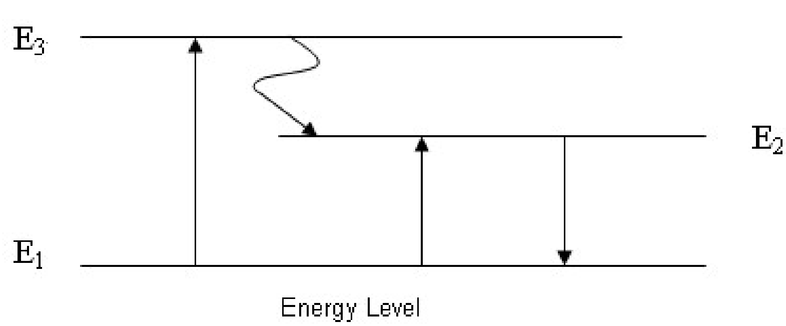

Figure 5 Energy diagram of laser gain medium

For example, for a three energy level system, the atoms are pumped from the ground state E_1 to the excited state E_3. The atom population at energy level E_3 rapidly drops to an intermediate energy level E_2 through a non-emission transition (transferring to thermal energy or crystal lattice vibration energy, but no photons are emitted). Because the intermediate energy level E_2 has a longer lifetime, the population of energy level E_2 will be accumulated. If the pumping light is strong enough, the population of E_1 will be reduced faster than the population decrease of E_2 back to E_1. Therefore, a population inversion is realized.

Laser generation must have a resonant cavity for providing optical positive feedback. Though atoms in an excited state will also spontaneously return to their ground state, spontaneously generated photons have random direction and phase. As a result, they will deviate from the optical axis and exit the cavity. Only stimulated emitted photons can travel along the optical axis. One portion of them will output through the cavity mirror and the other portion will be reflected back into the cavity and be amplified by the gain medium. Multiple reflections and amplifications within the cavity will generate enhanced stimulated emission — laser.

4.3 Optical Frequency Doubling

Optical frequency doubling is a common method to extend a waveband in nonlinear optics. Laser frequency doubling is a method that generates a frequency 2ω laser from a frequency ω laser source through the nonlinear effect of a crystal.

When light interacts with a medium, the electric dipoles of atoms will be generated due to the electromagnetic induction. The superposition of all diploes in unit volume is called the electric polarization intensity vector. This polarization field, ***P***, further emits electromagnetic radiation. When the electromagnetic field of external illumination is much weaker than the inner field of the substance, the induced polarization field is proportional to the external field intensity, *E*.

$$P = \varepsilon_0 \chi E \tag{1}$$

If light of different frequencies simultaneously illuminates a medium, it will be independently reflected, refracted or scattered. Hence, the linear superposition relationship is satisfied and no new frequency is generated. Once the external field is strong enough (e.g. laser illumination), the interaction between the laser light and the medium will result in a nonlinear response:

$$P = \alpha E + \beta E^2 + \gamma E^3 + \cdots \tag{2}$$

where, α, β, γ, … are the coefficients of the substance and their values reduce successively with an approximate ratio relationship:

$$\frac{\beta}{\alpha} = \frac{\gamma}{\beta} = \ldots = \frac{1}{E_{atom}} \tag{3}$$

where E_{atom} is the electric intensity inside the atom at an order of about 10^8 V/cm.

In the case of a weak external field, the nonlinear terms of E^2 and E^3 are all small quantities and are hence negligible; but when *E* is very strong, these nonlinear terms are not negligible.

Considering the square term of the field

$$E = E_0 \cos \omega t \tag{4}$$

$$P^{(2)} = \beta E^2 = \beta E_0^2 \cos^2 \omega t = \beta \frac{E_0^2}{2}(1 + \cos 2\omega t) \tag{5}$$

a frequency doubled term, $\cos 2\omega t$, exists. When a laser is incident on the frequency doubling crystal at a proper angle (matching angle), frequency doubled laser light can be generated.

The transfer efficiency of frequency doubled light with comparison to that of the fundamental frequency can be derived from nonlinear optics theory:

$$\eta = \frac{I_{2w}}{I_w} \propto \beta L^2 I_w \frac{\sin^2(\Delta kl/2)}{(\Delta kl/2)^2} \quad (6)$$

where l is the length of the frequency doubling crystal, I_w and I_{2w} are the intensities of the input fundamental frequency light and the output frequency doubled light, respectively. $\Delta k=2k_w-k_{2w}$ is the difference between the two light vectors k_w and k_{2w}.

It is apparent from equation (6) that a maximum transfer efficiency can be achieved only when Δk equals zero ($\Delta kl/2=0$), as

$$\Delta k = 2k_w - k_{2w} = \frac{4\pi}{\lambda_w}\left(n_w - n_{2w}\right) = 0 \quad (7)$$

Equation (7) means that maximum transfer efficiency is achieved only when the refractive indices of the fundamental frequency light and the frequency doubled light are equal. This is called the **phase matching condition**. Under normal dispersion, the refractive index of frequency doubled light n_{2w} is always larger than that of the fundamental light n_w, which results in phase mismatching. However, birefringent crystals have different refractive indices for ***O***-light and ***E***-light while the refractive index of ***E***-light varies with the angle change between the incident direction and the optical axis. Under normal dispersion, it is possible to compensate the refractive index difference between different light frequencies, so that the required phase matching can be realized.