4. Experimental Examples

4.1 Brewster's Angle

When unpolarized light travels from a transparent medium with a refractive index n_1 to another one with a higher refraction index n_2, part of the light is refracted into the second medium while the other part of the light is reflected back into the first medium. If the angles of incidence and refraction are θ_1 and θ_2, respectively, then the following condition exists, known as Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{2}$$

According to Sir David Brewster, at a specific angle of incidence, θ_b, called Brewster's angle, the reflected and the refracted rays are perpendicular to each other, so the sum of the incident angle and the refractive angle is 90° as

$$\theta_b + \theta_2 = 90°\text{, namely } \theta_2 = 90° - \theta_b \tag{3}$$

By substituting equation (3) into equation (2), we get

$$\begin{aligned} n_1 \sin\theta_b &= n_2 \sin(90° - \theta_b) = n_2 \cos\theta_b \\ \tan\theta_b &= \frac{n_2}{n_1} \end{aligned} \tag{4}$$

Equation (4) is known as Brewster's Law. When the incident beam is travelling in air, $n_1 \approx 1.00$, equation (4) becomes $\tan\theta_b = n_2$. The Brewster's angle of a material is sometimes referred to as the polarizing angle of the material.

Note: Using Brewster's Law, one can determine the refractive index of a glass sheet. This can be achieved by substituting the value of the Brewster's angle into Eq. (4) to obtain n_2.

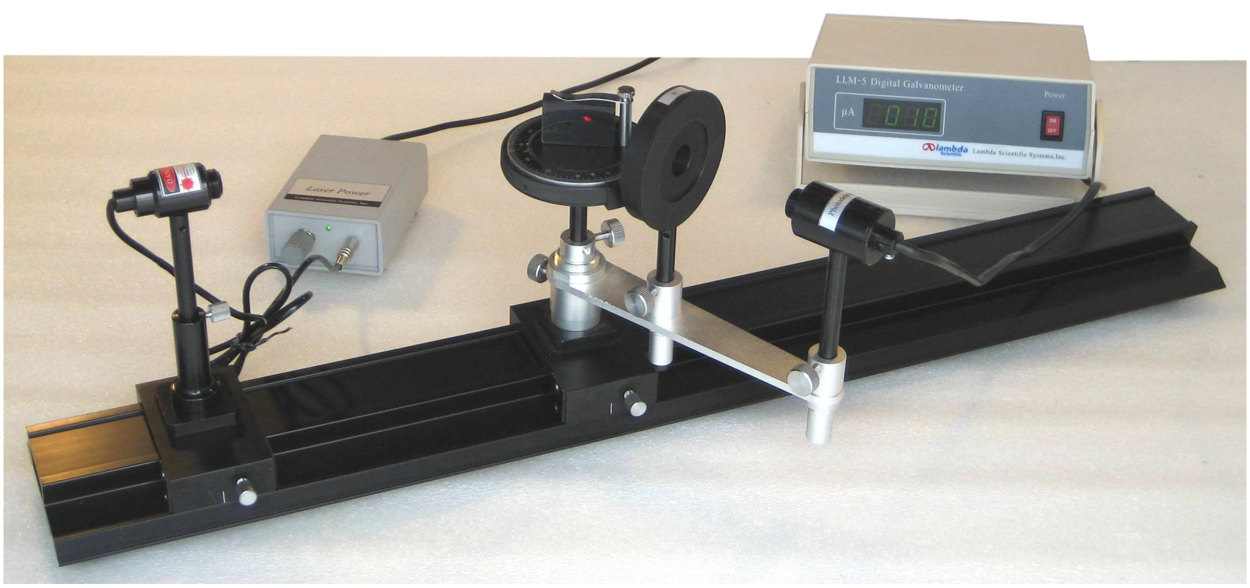

Figure 1 Experimental setup for measurement of Brewster's angle

Experimental Procedure:

1. Per Figure 1, place one general carrier and the special carrier (for mounting rotation stage) on the rail. Mount the rotation stage onto the special carrier.
2. Mount the photo detector into the last hole and the polarizer into the middle hole of the rotation arm.
3. Mount the diode laser onto the general carrier. Turn on the laser and adjust the height of laser beam to the center of the polarizer. Laser power can be adjusted with the potentiometer on the Laser Power box.
4. Place the sample glass block onto the stage (i.e. insert the positioning pin of the block into the hole near the stage center), let its reflection surface pass through the rotation center (i.e. along the 0°-180° line), and fix it using the press clamp.
5. Rotate the stage, let the incident laser beam shine perpendicularly to the reflection surface of the sample. Record the angle reading of the stage as i_0.
6. Rotate the stage by about 56°~57º, and rotate the rotation arm to follow the reflected laser beam and let it hit the center of the active area of the detector.
7. Carefully turn the stage and the rotating arm, while rotating the polarization direction of the polarizer until a minimum reading is observed on the galvanometer. Record the angle reading on the stage as i_1. Repeat 3 times of the measurement to get an average of angle i_1. Then Brewster's angle is $i = i_1 - i_0$. (refer to Figure 2).

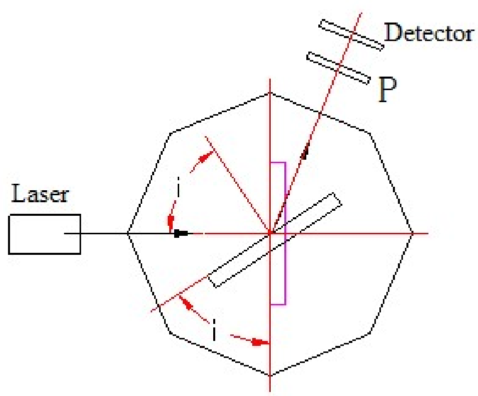

Figure 2 Schematic of experimental configuration

4.2 Verification of Malus's Law

Malus's law defines the transmitted intensity of light through two polarizers with an angle of θ between the transmission directions of the polarizers, as

$$I = I_{max} \cos^2 \theta \qquad (5)$$

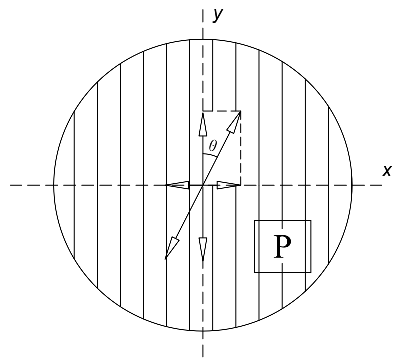

Figure 3 Schematic of Malus's Law

The maximum and minimum transmission occurs when θ = 0º and θ = 90º, respectively.

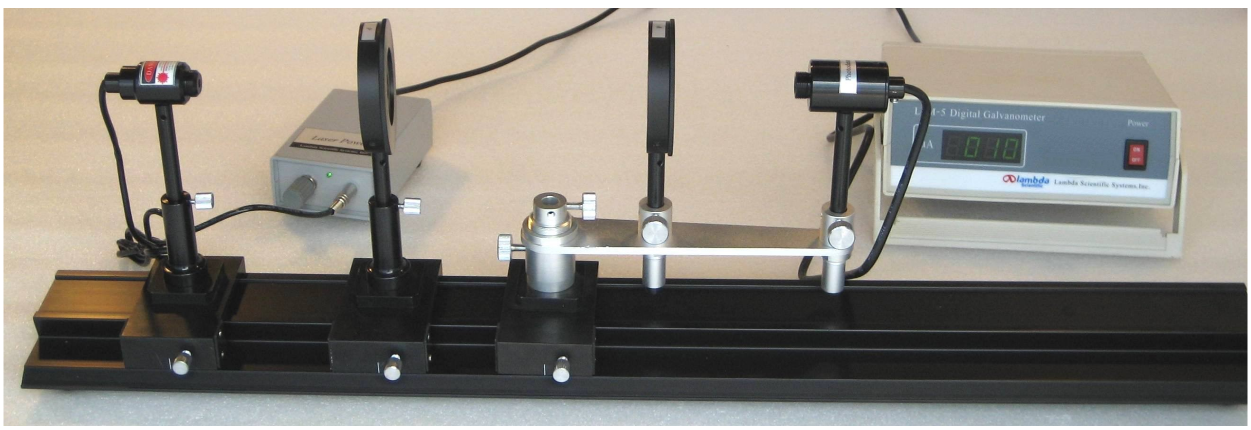

Figure 4 Experimental setup for verification of Malus's Law

Experimental Procedure:

1. Per Figure 4, mount the diode laser and one polarizer on a general carrier, respectively. Mount the special carrier on the rail.
2. Mount the photo detector into the last hole of the rotation arm.
3. Adjust the height of laser beam to the center of the polarizer. Rotate the polarizer with its rotary holder until the transmitted laser intensity reaches a maximum value on the photo detector.
4. Place the post of the analyser (another polarizer) into the middle hole of the rotation arm. Adjust the height of the analyser to that of the polarizer.
5. Rotate the analyser until the transmitted laser intensity reaches a minimum value on the photo detector. Now the axes of the polarizer and the analyser are perpendicular to each other.
6. Continue to rotate the analyser until the transmitted laser intensity reaches another minimum value. Check the angular reading of the analyzer, it should be rotated by 180° from the previous angle.

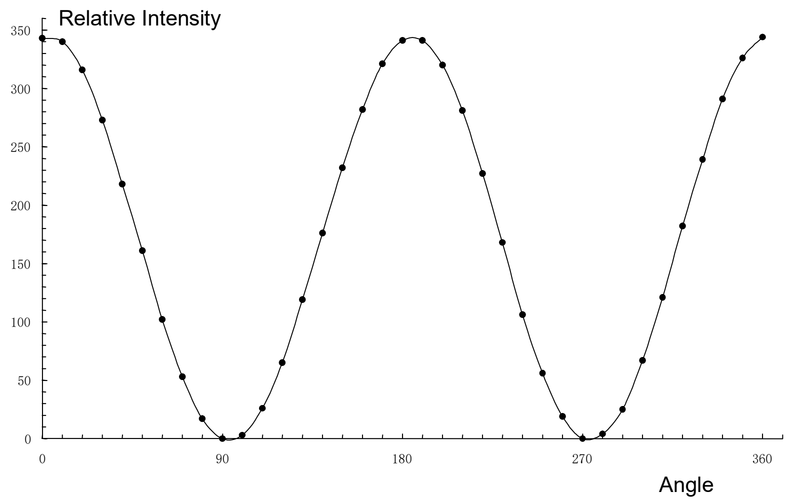

Figure 5 Plot of transmitted intensity vs relative angle

7. Rotate the second polarizer (i.e. analyser) by 10º and read the transmitted intensity of the laser.
8. Repeat Step 7 and collect intensity readings for polarizer orientations between 0º and 360º in steps of 10º in the same direction.
9. Draw a curve of intensity vs relative angle, as shown in Figure 5.

4.3 Function of Half-Wave Plate

When a light beam passes through a birefractive crystal, it is divided into two polarized light beams, one is called the ordinary light ***O*** because it obeys the 'Law of Refraction'; the other beam is called the extraordinary light ***E***, as it does not obey the 'Law of Refraction'. Because ***O*** light travels in a birefractive crystal with a thickness ***d*** at a different speed from ***E*** light, there will be a phase difference between the two light beams as

$$\delta = \frac{2\pi}{\lambda}\left(n_0 - n_e\right)d \tag{6}$$

where λ is the wavelength of light, n_0 is the refractive index of ***O*** light, and n_e is the refractive index of ***E*** light. A retardation plate whose thickness makes $\delta=(2k+1)\pi$, $k=0, \pm1, \pm2\ldots$ is called a half-wave plate while a retardation plate whose thickness makes $\delta=(2k+1)\pi/2$, $k=0, \pm1, \pm2\ldots$ is called a quarter-wave plate.

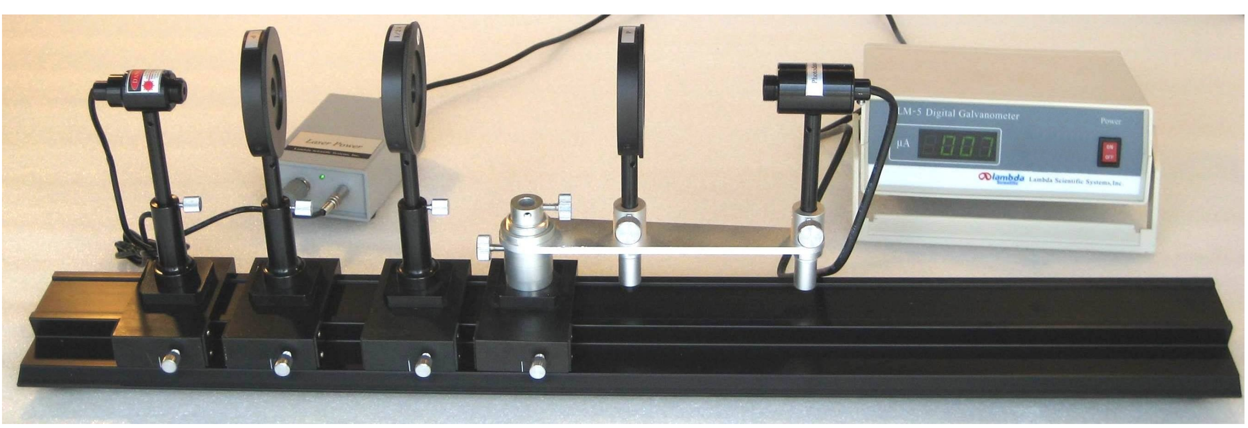

Figure 6 Experimental setup with half-wave plate

Experimental Procedure:

1. Per Figure 6, mount the diode laser and one polarizer on a general carrier, respectively. Mount the special carrier on the rail.
2. Mount the photo detector into the last hole of the rotation arm.
3. Adjust the height of laser beam to the center of the polarizer. Rotate the polarizer with its rotary holder until the transmitted laser intensity reaches a maximum value on the photo detector.
4. Place the post of the analyser (another polarizer) into the middle hole of the rotation arm. Adjust the analyser to the height of the polarizer.
5. Rotate the analyser until the transmitted laser intensity reaches a minimum value as on the photo detector. Now the axes of the polarizer and the analyser are perpendicular to each other.
6. Mount the half-wave plate onto a general carrier. Place it between the two polarizers.
7. Rotate the half-wave plate to observe the intensity change of the detected light.
8. Rotate the half-wave plate until the detected light intensity reaches minimum. Next, rotate the half-wave plate by 15º while rotating the analyser to achieve minimum detection. Record the angular change of polarization axis of the analyser.
9. Repeat Step 8 in step of 15º while recording the corresponding angle change of the analyser.

4.4 Quarter-Wave Plate: Circularly and Elliptically Polarized Light

If right (*R*) and a left (*L*) circularly polarized waves of equal amplitude, are superimposed, the result is a plane-polarized wave. Conversely, a plane-polarized light wave can be decomposed into *R* and *L* components. If the amplitudes of the two circularly polarized waves are not equal, the tip of the resultant ***E*** vector follows an elliptical path and such light is said to be elliptically polarized.

Experimental Procedure:

The steps of this experiment are quite similar to those in 4.3. Please use the quarter-wave plate to replace the half-wave plate. The schematic of a quarter-wave plate is shown in Figure 7.

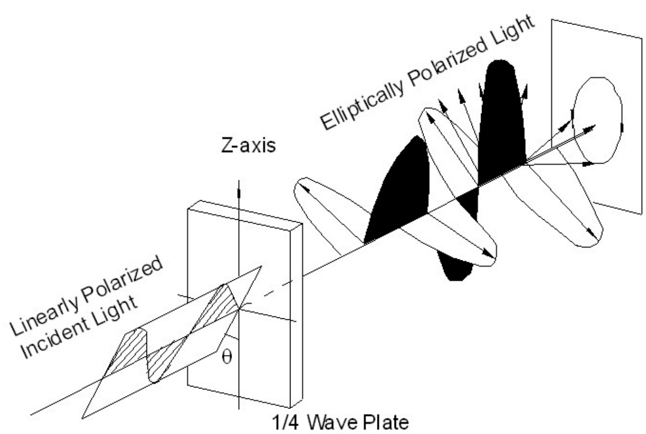

Figure 7 Principle of quarter-wave plate

4.5 Effect of Optical Activity

When linearly polarized light passes through certain solid substances or solutions, the plane of polarization of the light rotates by a certain angle. This phenomenon is called the effect of optical activity and the optical rotatory angle is called the specific rotation of the substance. The specific rotation of a solution depends on a number of parameters such as the substance in the solution, the concentration of solution, the sample path length, the temperature of solution, and the wavelength of light. If other parameters are fixed, then the specific rotation, θ, is linearly proportional to the concentration of solution, C, as

$$\theta = \beta C \tag{7}$$

where β depends on the substance in the solution, the sample path length, the temperature of solution, and the wavelength of light.

The polarization rotation ability of a substance can be further evaluated by its specific rotatory power, as described by

$$[a]_{\lambda}^{T} = \frac{\theta}{lC} \tag{8}$$

where T represents the temperature of the solution (°C), λ is the wavelength of monochromatic light (nm), θ is the specific rotation degree, l is the sample path length (dm), and C is the concentration of solution (g/100 mL).

It is apparent from equation (8) that **a)** the plane of polarization of light rotates gradually as light propagates in the solution so that the specific rotation is proportional to the length of the sample, **b)** the specific rotation is also proportional to the concentration of the solution.

If the concentration of the solution and the sample path length are known, the specific rotatory power of the solution can be calculated once the specific rotation is measured. This can be conducted by measuring the specific rotation while varying the solution concentration. From the slope of the obtained θ-C line, the specific rotatory power of the substance can be derived. Similarly, if the specific rotatory power of a substance is known, then its concentration in a solution under test can be determined once the specific rotation is measured.

The specific rotation of an optically active medium can be either left-handed or right-handed. When viewed against the propagation direction of light, if the specific rotation is clockwise, then the substance is called a right-handed substance; if the specific rotation is counter-clockwise, then the substance is called a left-handed substance.

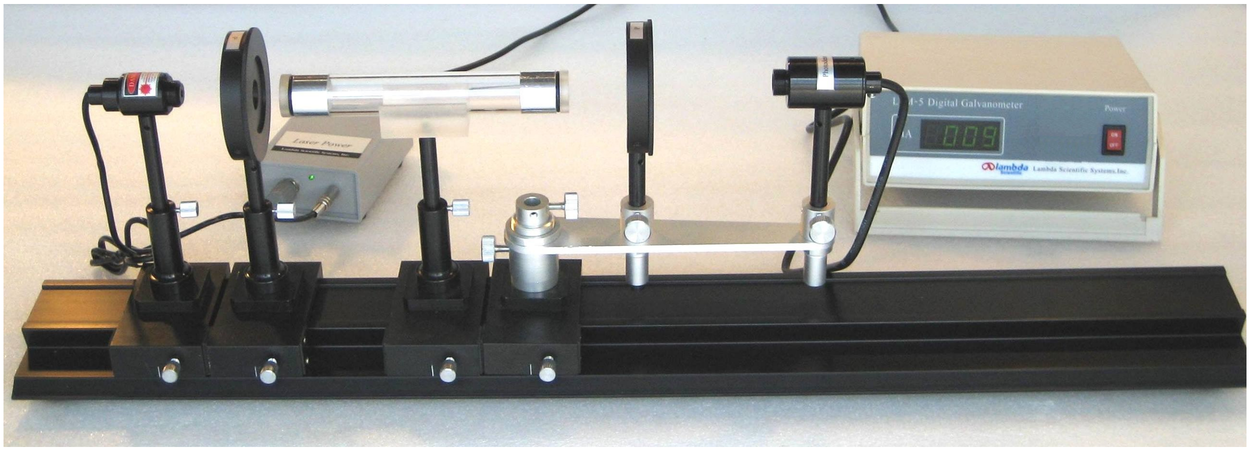

Figure 8 Experimental setup of optical activity

4.5.1 Observe the Polarization Rotation Characteristics of Glucose Solution

Experimental Procedure:

1. Per Figure 8, mount the diode laser and one polarizer on a general carrier, respectively. Mount the special carrier on the rail.
2. Mount the photo detector into the last hole of the rotation arm.
3. Adjust the height of laser beam to the center of the polarizer. Rotate the polarizer with its rotary holder until the transmitted laser intensity reaches a maximum value on the photo detector.
4. Place the post of the analyser (another polarizer) into the middle hole of the rotation arm. Adjust the analyser to the height of the polarizer.
5. Rotate the analyser until the transmitted laser intensity reaches a minimum value on the photo detector. Now the axes of the polarizer and the analyser are perpendicular to each other.
6. Add glucose solution into the sample cell, place the sample cell (with a carrier) on the rail between the polarizer and the analyser, per Figure 8. Note: (1) if the solution is in the cell for a long time, shake the sample cell before mounting it to carrier, (2) let the two polarizers to the ends of the cell as closely as possible.
7. Adjust the height of the sample cell to make the laser beam pass through the sample cell. Note: the laser beam should pass the ends of the sample cell perpendicularly.
8. Rotate the analyser carefully and slowly until the transmitted light intensity reaches a minimum value again to judge if the polarization rotation of the glucose solution is left-handed or right-handed.
9. Record the angle rotated by the analyser as the specific rotation angle of the glucose solution under test.

4.5.2 Measure the Specific Rotatory Power of Glucose Solution

Experimental Procedure:

1. Make glucose solution with five different concentrations of 30% (C_0), 15% ($C_0/2$), 7.5% ($C_0/4$), 3.75% ($C_0/8$), and 0%, respectively.

2. Measure the specific rotation angle of each glucose solution (for each glucose solution, repeat the measurement multiple times and then take the averaged data).
3. Record the ambient temperature during each measurement. Plot the curve of specific rotation angle versus solution concentration using the following table.
4. Calculate the slope of the measured curve by curve-fitting to derive the specific rotatory power of the glucose solution.

	Specific Rotation Angle $\theta(°)$						$\bar{\theta}(°)$
	1	2	3	4	5	6	
C_0							
$C_0/2$							
$C_0/4$							
$C_0/8$							
0（Pure water）							

4.5.3 Measure the Concentration of Glucose Solution under Test

Per 4.5.2, if the specific rotatory power of glucose solution is known, then the concentration of a glucose solution under test can be determined once its specific rotation is measured.