4. Experimental Examples

4.1 Brewster's Angle

When unpolarized light travels from a transparent medium with a refractive index n_1 to another one with a higher refraction index n_2, part of the light is refracted into the second medium while the other part of the light is reflected back into the first medium. If the angles of incidence and refraction are θ_1 and θ_2, respectively, then the following condition exists, known as Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{2}$$

According to Sir David Brewster, at a specific angle of incidence, θ_b, called Brewster's angle, the reflected ray and the refracted ray are perpendicular to each other, so the sum of the incident angle and the refractive angle is 90° as

$$\theta_b + \theta_2 = 90°\text{, namely } \theta_2 = 90° - \theta_b \tag{3}$$

By substituting equation (3) into equation (2), we get

$$\begin{gathered} n_1 \sin\theta_b = n_2 \sin(90° - \theta_b) = n_2 \cos\theta_b \\ \tan\theta_b = \frac{n_2}{n_1} \end{gathered} \tag{4}$$

Equation (4) is known as Brewster's Law. When the incident beam is travelling in air, $n_1 \approx 1.00$, equation (4) becomes $\tan\theta_b = n_2$. The Brewster's angle of a material is sometimes referred to as the polarizing angle of the material.

Note: Using Brewster's Law, one can determine the refractive index of a black glass. This can be achieved by substituting the value of the Brewster's angle into Eq. (4) to obtain n_2.

Figure 1 Experimental setup for measurement of Brewster's angle

Experimental Procedure:

1. Mount the laser tube onto the laser holder and connect the power supply.

2. Turn on the laser. Change the laser height and inclination by adjusting the six screws of the laser tube holder, so that the laser beam is parallel to the rail surface and propagates along its center.

3. Insert the goniometer into the post holder on carrier and secure the carrier on the other end of the optical rail. Unpack the black glass and secure it on the center of the dial using the clamping arm. Make sure one edge of the glass just overlaps the 90º axis of the dial as shown in Figure 2. Adjust tilt of the black glass to let the reflected laser beam again parallel to the rail. Secure the black glass.

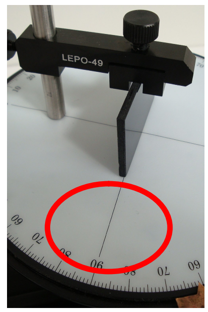

Figure 2 Black glass mounted on goniometer

> **Note:**
> Handle the black glass with care. Keep the two reflective surfaces from fingerprints and dirt.

4. Mount a polarizer into the polarizer holder, place it onto a carrier, and locate the carrier next to the laser. Rotate the polarizer to set its axis along horizontal direction so that the transmitted light is horizontally polarized. Insert detector on the hole next to the dial of the goniometer and connect the detector to the amplifier. Make sure the detector area faces the point where two axes of the dial intersect.

5. Set the pointer of goniometer at 56°~57º and rotate the dial to allow the reflected laser beam to hit the center of the active area of the detector. Read the light intensity on the digital display of the photocurrent amplifier.

6. Carefully turn the rotating arm and the dial to achieve the minimum readout. Record the reading on the dial. This is the Brewster's angle.

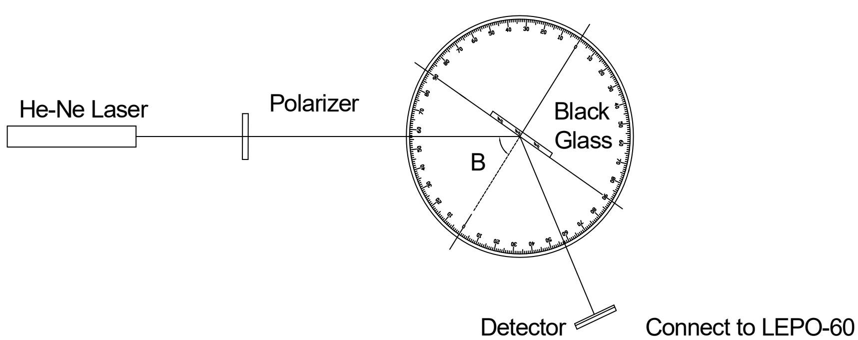

Figure 3 Schematic of experimental configuration

4.2 Verification of Malus's Law

Malus's law defines the transmitted intensity of light through two polarizers with an angle of θ between the transmission directions of the polarizers, as

$$I = I_{max} \cos^2 \theta \tag{5}$$

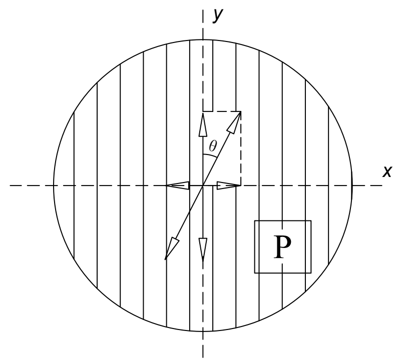

Figure 4 Schematic of Malus's Law

The maximum and minimum transmission occurs when $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively.

Figure 5 Experimental setup for verification of Malus's Law

Experimental Procedure:

1. Mount the laser tube onto the laser holder and connect the power supply.

2. Mount the polarizer into the polarizer holder and place it onto a carrier, and rotate the polarizer so that only horizontally polarized light can pass through (the short line on polarizer is parallel to the optical axis).

3. Repeat Step 2 with another polarizer, holder and carrier, and place the assembly next to the first polarizer.

4. Secure the optical detector in carrier and connect the detector to the photocurrent amplifier. Turn on the amplifier. Make sure the laser beam hits the central active area of the detector head.

5. Record the relative intensity of the polarized laser.

6. Rotate the second polarizer by 10º and read the transmitted intensity of the laser.

7. Repeat Step 6 and collect intensity readings for polarizer orientations between 0º and 360 º in steps of 10º in the same direction.

8. Draw a curve of intensity vs. relative angle, as shown in the following plot.

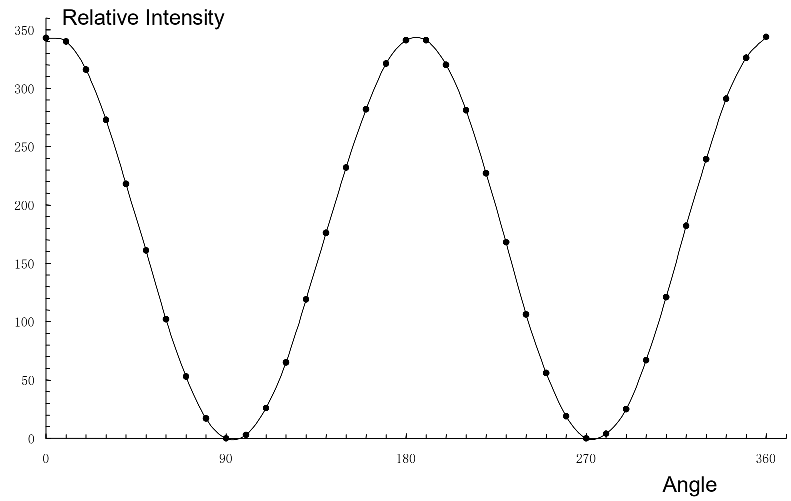

Figure 6 Plot of transmitted intensity vs relative angle

4.3 Function of Half-Wave Plate

When a light beam passes through a birefractive crystal, it is divided into two polarized light beams, one is called the ordinary light ***O*** because it obeys the 'Law of Refraction'; the other beam is called the extraordinary light ***E***, as it does not obey the 'Law of Refraction'. Because ***O*** light travels in a birefractive crystal with a thickness ***d*** at a different speed from ***E*** light, there will be a phase difference between the two light beams as

$$\delta = \frac{2\pi}{\lambda}(n_0 - n_e)d \tag{6}$$

where λ is the wavelength of light, n_0 is the refractive index of ***O*** light, and n_e is the refractive index of ***E*** light. A retardation plate whose thickness makes $\delta=(2k+1)\pi$, $k=0, \pm1, \pm2\ldots$ is called a half-wave plate while a retardation plate whose thickness makes $\delta=(2k+1)\pi/2$, $k=0, \pm1, \pm2\ldots$ is called a quarter-wave plate.

Figure 7 Experimental setup with half-wave plate

Experimental Procedure:

1. Mount the laser tube onto the laser holder and connect the power supply.

2. Mount the polarizer into the polarizer holder and place it onto the carrier, and rotate the polarizer so that only horizontally polarized light can pass through.

3. Repeat Step 2 with another polarizer, holder and carrier, and place it about 25 mm away from the first polarizer.

4. Hold a piece of blank paper next to the second polarizer. A light spot should be observed on the paper. Rotate the polarizer till the light spot disappears.

5. Assemble beam expander and lens holder with carrier. Place them next to the first polarizer. Adjust the beam expander so that the expanded light illuminates the second polarizer. Use the collimating lens if necessary behind the beam expander to control the beam size of the laser beam.

6. Assemble half-wave plate, polarizer holder and carrier. Place them between the beam expander and the second polarizer.

7. Rotate the half-wave plate through 360º to observe the brightness change of the light spot.

8. Rotate the half-wave plate till light spot on the paper disappears. Then, rotate the wave plate by 15º and rotate the polarizer until the light spot disappears. Record the angular change of polarization axis of the polarizer.

9. Repeat Step 8 in increments of 15º. Record every angle change of the polarizer.

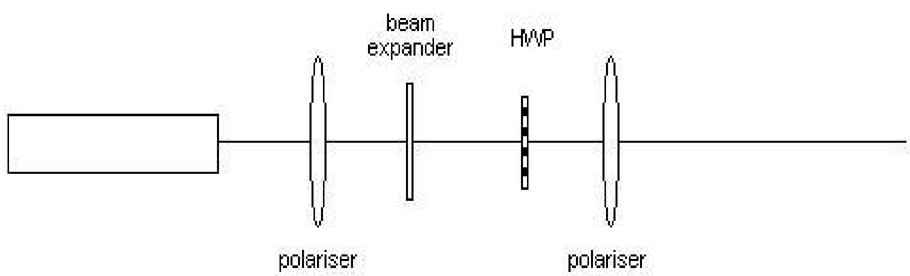

Figure 8 Schematic of half-wave plate experiment

4.4 Quarter-Wave Plate: Circularly and Elliptically Polarized Light

If a right (*R*) and a left (*L*) circularly polarized wave, both of equal amplitude, are superimposed, the result is plane-polarized light. Conversely, plane-polarized light can be decomposed into *R* and *L* components. If the amplitudes of the two circularly polarized waves are not the same, the tip of the resultant ***E*** vector will follow an elliptical path and such light is said to be elliptically polarized.

Experimental Procedure:

The steps of this experiment are quite similar to those in 4.3. Please use the quarter-wave plate to replace the half-wave plate. By properly arranging the sequence and axis orientation of the relevant optical components, a linearly polarized beam can be converted to a circularly polarized beam, or vice versa.

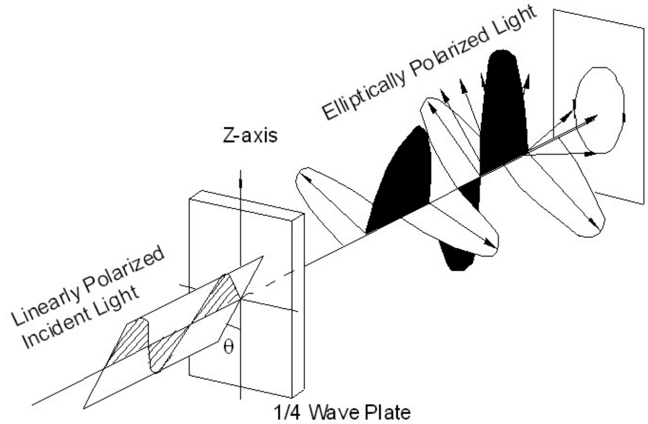

Figure 9 Principle of quarter-wave plate

Notes:

a) The axis direction of a ¼ λ wave plate is marked on its mount. However, the actual axis direction may be deviated from the mark line. To determine the actual axis direction, place the ¼ λ wave plate in between two crossed polarizers which is illuminated by the He-Ne laser. Rotate the ¼ λ wave plate till the minimum laser output is achieved. At this time, the axis of the ¼ λ wave plate is parallel to the axis of either of the two polarizers.

b) An ideal ¼ λ waveplate would produce a π/2 (90°) phase difference between the two orthogonal lightwave components. Due to manufacturing errors, the actual phase difference may deviate from π/2. In this case, the use of a non-ideal ¼ λ waveplate does not produce a perfectly circularly polarized beam, but an elliptically polarized beam. In this experimental setup, the ¼ λ waveplate is educational grade, and the phase difference error can reach 5%. By analyzing the output polarization state (i.e. ellipticity and axis direction), the actual phase difference value can be obtained.