4. Experimental Examples

4.1 Brewster’s Angle

When unpolarized light travels from a transparent medium with a refractive index \( n_1 \) to another one with a higher refraction index \( n_2 \), part of the light is refracted into the second medium while the other part of the light is reflected back into the first medium. If the angles of incidence and refraction are \( \theta_1 \) and \( \theta_2 \), respectively, then the following condition exists, known as Snell's law

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2 \tag{2}
\]

According to Sir David Brewster, at a specific angle of incidence, \( \theta_b \), called Brewster’s angle, the reflected ray and the refracted ray are perpendicular to each other, so the sum of the incident angle and the refractive angle is 90° as

\[
\theta_b + \theta_2 = 90°, \text{ namely } \theta_2 = 90° - \theta_b \tag{3}
\]

By substituting equation (3) into equation (2), we get

\[
n_1 \sin \theta_b = n_2 \sin(90° - \theta_b) = n_2 \cos \theta_b
\]

\[
\tan \theta_b = \frac{n_2}{n_1} \tag{4}
\]

Equation (4) is known as Brewster's Law. When the incident beam is travelling in air, \( n_1 \approx 1.00 \), equation (4) becomes \( \tan \theta_b = n_2 \). The Brewster’s angle of a material is sometimes referred to as the polarizing angle of the material.

**Note:** Using Brewster’s Law, one can determine the refractive index of a black glass. This can be achieved by substituting the value of the Brewster’s angle into Eq. (4) to obtain \( n_2 \).

![Figure 1 Experimental setup for measurement of Brewster’s angle](image)

**Experimental Procedure:**

1. Mount the laser tube onto the laser holder and connect the power supply.
2. Turn the laser on and slide the alignment aperture along the rail. Mount a polarizer onto the polarizer holder, place it onto a carrier next to the laser. Rotate the polarizer to set its axis along horizontal direction so that the transmitted laser beam is horizontally polarized. Since the laser is
linearly polarized, it should be rotated so that the laser light transmitted through the polarized is maximized.

3. Place the alignment aperture onto a carrier and slide it back and forth along the rail while altering the laser height and tilt by adjusting the six screws of the laser holder to allow the laser beam to pass through the aperture along the rail. **Note**: do not rotate the laser tube anymore.

4. Remove the alignment aperture from the rail, insert the goniometer into the post holder on carrier and secure the carrier on the other end of the optical rail. Unpack the black glass and secure it on the center of the dial using the clamping arm. Make sure one edge of the glass just overlaps the 90° axis of the dial as shown in Figure 2. Adjust tilt of the black glass to let the reflected laser beam pass through the alignment aperture along the rail. Secure the black glass.

![Figure 2 Black glass mounted on goniometer](image)

**Note:**
Handle the black glass with care. Keep the two reflective surfaces from fingerprints and dirt.

5. Insert the detector on the hole next to the dial of the goniometer and connect the detector to the amplifier. Make sure the detector area faces the point where two axes of the dial intersect.

6. Set the pointer of goniometer at 56°~57° and rotate the dial to allow the reflected laser beam to hit the center of the active area of the detector. Read the light intensity on the digital display of the photocurrent amplifier.

7. Carefully turn the rotating arm and the dial to achieve the minimum readout. Record the reading on the dial. This is the Brewster’s angle.

![Figure 3 Schematic of experimental configuration](image)
4.2 Verification of Malus's Law

Malus's law defines the transmitted intensity of light through two polarizers with an angle of $\theta$ between the transmission directions of the polarizers, as

$$I = I_{\text{max}} \cos^2 \theta$$  \hspace{1cm} (5)

The maximum and minimum transmission occurs when $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively.

**Experimental Procedure:**

1. Repeat steps 1 to 3 in experiment 4.1.
2. Place another polarizer after the first polarizer on the rail, and align the polarization axis of the second polarizer parallel to that of the first polarizer.
3. Secure the optical detector in carrier and connect the detector to the photocurrent amplifier. Turn on the amplifier. Make sure the laser beam hits the central active area of the detector head.
4. Record the relative intensity of the polarized laser.
5. Rotate the second polarizer by $10^\circ$ while recording the transmitted intensity of the laser.
6. Repeat Step 5 and collect intensity readings for polarizer orientations between $0^\circ$ and $360^\circ$ in steps of $10^\circ$ in the same direction.
7. Draw a curve of intensity versus relative angle, as shown in the following plot.
4.3 Function of Half-Wave Plate

When a light beam passes through a birefractive crystal, it is divided into two polarized light beams, one is called the ordinary light $O$ because it obeys the ‘Law of Refraction'; the other beam is called the extraordinary light $E$, as it does not obey the 'Law of Refraction'. Because $O$ light travels in a birefractive crystal with a thickness $d$ at a different speed from $E$ light, there will be a phase difference between the two light beams as

$$\delta = \frac{2\pi}{\lambda}(n_0 - n_e)d$$  

(6)

where $\lambda$ is the wavelength of light, $n_0$ is the refractive index of $O$ light, and $n_e$ is the refractive index of $E$ light. A retardation plate whose thickness makes $\delta=(2k+1)\pi, k=0, \pm1, \pm2\ldots$ is called a half-wave plate while a retardation plate whose thickness makes $\delta=(2k+1)\pi/2, k=0, \pm1, \pm2\ldots$ is called a quarter-wave plate.

Experimental Procedure:

1. Repeat steps 1 to 3 in experiment 4.1.
2. Place another polarizer about 25 mm away from the first polarizer on the rail, and align the polarization axis of the second polarizer parallel to that of the first polarizer.
3. Hold a piece of blank paper next to the second polarizer. A light spot should be observed on the paper. Rotate the second polarizer till the light spot disappears.
4. Assemble the beam expander onto a lens holder with carrier, and place it next to the first polarizer. Adjust the beam expander so that the expanded light illuminates the second polarizer. Use the collimating lens \( f=150 \text{ mm} \) if necessary behind the beam expander to control the beam size of the laser beam.

5. Assemble the half-wave plate onto a polarizer holder with carrier, and place it between the beam expander and the second polarizer, as shown in Figure 8.

6. Rotate the half-wave plate through 360° to observe the brightness change of the light spot on a piece of white paper placed after the second polarizer.

7. Rotate the half-wave plate till light spot on the paper disappears. Then, rotate the half-wave plate by 15° and rotate the second polarizer until the light spot disappears again. Record the angular change of polarization axis of the polarizer.

8. Repeat Step 7 in step of 15° while recording the corresponding angle of the second polarizer.

4.4 Quarter-Wave Plate: Circularly and Elliptically Polarized Light

If right (R) and left (L) circularly polarized waves of equal amplitude, are superimposed, the result is plane-polarized light. Conversely, plane-polarized light can be decomposed into R and L components. If the amplitudes of the two circularly polarized waves are not the same, the tip of the resultant \( E \) vector follows an elliptical path and such light is said to be elliptically polarized.

**Experimental Procedure:**

The steps of this experiment are quite similar to those in 4.3. Please use the quarter-wave plate to replace the half-wave plate.