2. Theory

When ultrasound waves travel in a medium, the medium is subject to elastic strain with periodic changes in both time and space, causing a similar periodical change in the refractive index of the medium. As a result, when a ray of light passes through a medium in the presence of ultrasound waves in the medium, it is diffracted by the medium acting as a phase grating. This is the basic theory of acousto-optic effect.

Acousto-optic effect is classified into normal acousto-optic effect and anomalous acousto-optic effect. In an isotropic medium, the plane of polarization of the incident light is not changed by the acousto-optic interaction (called normal acousto-optic effect); in an anisotropic medium, the plane of polarization of the incident light is altered by the acousto-optic interaction (called anomalous acousto-optic effect). Anomalous acousto-optic effect provides the key foundation for the fabrication of advanced acousto-optic deflectors and tunable acousto-optic filters. Unlike normal acousto-optic effect, anomalous acousto-optic effect cannot be explained by Raman-Nath diffraction. However, by using parametric interaction concepts such as momentum matching and mismatching in nonlinear optics, a unified theory of acousto-optic interaction can be established to explain both normal and anomalous acousto-optic effects. The experiments in this system only cover normal acousto-optic effect in isotropic media.

As shown in Figure 1, sound waves with a wavelength of λ_s travel in an acousto-optic medium along the y-axis at an angular frequency of ω_s with a wave vector of k_s, while a ray of light with a wavelength of λ is incident on the medium along the x-axis at an angular frequency of ω with a wave vector of k. Because the speed of light in the medium is 10^5 times faster than that of the sound wave, the periodical change in space of the medium can be considered stationary. The change in the optical indicatrix of the medium due to elastic strain is given by

$$\Delta B = PS \tag{1}$$

where S is the strain, and P is the photo-elastic coefficient of the medium. Normally, P and S are the second-order tensors, but they can be treated as scalars when dealing with isotropic media. Similarly, strain also propagates in the medium as a traveling wave, described by

$$S = S_0 \sin(\omega_s t - k_s y) \tag{2}$$

If strain is small, the refractive index of the medium can be written as

$$n(y,t) = n_0 + \Delta n \sin(\omega_s t - k_s y) \tag{3}$$

where n_0 is the refractive index of the medium in the absence of ultrasound waves, and Δn is the amplitude of variation in the refractive index generated by the acoustic wave, given as

$$\Delta n = -\frac{1}{2} n^3 P S_0 \tag{4}$$

Because light wave propagates perpendicularly with respect to the sound wave ($k \perp k_s$) to pass through a medium with a thickness of L, a phase delay caused by the medium is

$$\Delta\phi = k_0 n(y,t)L = k_0 n_0 L + k_0 \Delta n L \sin(\omega_s t - k_s y) = \Delta\phi_0 + \delta\phi \sin(\omega_s t - k_s y) \tag{5}$$

Where k_0 is the wave vector of light in vacuum, $\Delta\phi_0$ is the phase delay caused by the medium in the absence of ultrasound, and the second term is the added phase delay caused by the ultrasound (phase modulation). Obviously, the wavefront of the incident light is altered by the ultrasound to become wrinkled wavefront, hence the propagation characteristic of the light is altered by the medium in the presence of ultrasound. As a result, diffraction of light occurs.

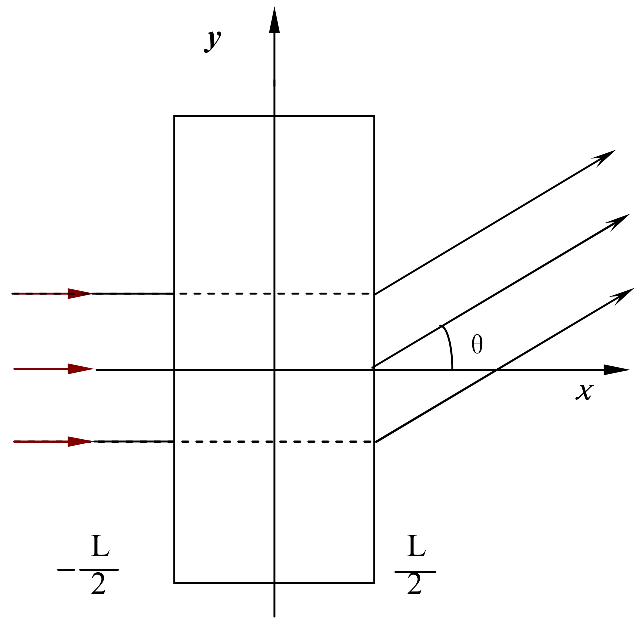

Figure 1 Schematic of acousto-optic diffraction

If the oscillation of the electric field vector of light at the incident plane (x=-L/2) is $E_i = Ae^{it}$, then the superposition of diffracted light exiting from the exit plane (x=L/2) at a location on the xy plane but far from the exit plane can be expressed as

$$E \propto A \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i[(\omega t - k_0 n(y,t)L) - k_0 y \sin\theta]} dy \tag{6}$$

Equation (6) can be further rewritten as

$$E = Ce^{i\omega t} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i\delta\phi \sin(k_s y - \omega_s t)} e^{-ik_0 y \sin\theta} dy \tag{7}$$

where b is the diameter of the light beam, θ is the diffraction angle, C is a constant related to the amplitude of the incident wave.

By using the expression for Bessel functions

$$e^{ia\sin\theta} = \sum_{m=-\infty}^{\infty} J_m(a)e^{im\theta} \tag{8}$$

Equation (7) can be further expressed as

$$E = Cb\sum_{m=-\infty}^{\infty} J_m(\delta\phi)e^{i(\omega-m\omega_s)t}\frac{\sin[b(mk_s - k_0\sin\theta)/2]}{b(mk_s - k_0\sin\theta)/2} \tag{9}$$

The term related to the m^{th}-order diffraction in equation (9) is

$$E_m = E_0 e^{i(\omega-m\omega_s)t} \tag{10}$$

$$E_0 = CbJ_m(\delta\phi)\frac{\sin[b(mk_s - k_0\sin\theta)/2]}{b(mk_s - k_0\sin\theta)/2} \tag{11}$$

Because the sinc function $\sin(x)/x$ is maximum when x=0, the diffraction angle of the m^{th}-order maximum diffraction is determined by

$$\sin\theta_m = m\frac{k_s}{k_0} = m\frac{\lambda_0}{\lambda_s} \tag{12}$$

where λ_0 is the wavelength of light in vacuum.

Obviously, the medium with strain caused by ultrasound waves acts as a diffraction grating with a grating period equal to the wavelength of the ultrasound in the medium. It is known from equation (10) that the angular frequency of the m^{th}-order diffracted light is

$$\omega_m = \omega - m\omega_s \tag{13}$$

Thus, the diffracted light is still monochromatic, but with a slightly shifted frequency ($\omega >> \omega_s$).

The maximum intensity of the m^{th}-order diffracted light is given by

$$I_m = E_m E_m^* = E_0 E_0^* = C^2 b^2 J_m^2(\delta\phi) = I_0 J_m^2(\delta\phi) \tag{14}$$

The diffraction efficiency (η_m) of the m^{th}-order diffracted light is defined as the ratio of the intensity of the m^{th}-order diffracted light to that of incident light. It is apparent from equation (14) that η_m is proportional to the square of $J_m(\delta\phi)$. If m is an integer, then $J_{-m}(a)=(-1)^m J_m(a)$. Therefore, the positive- and negative-order diffracted light is distributed symmetrically around the 0^{th}-order diffracted light (undiffracted).

If light is incident on the medium with an angle and the length of acousto-optic interaction is less than $(\lambda_s)^2/2\lambda_0$, then diffraction angle of the m^{th}-order maximum diffraction is given by

$$\sin\theta_m = \sin\theta_i + m\frac{\lambda_0}{\lambda_s} \quad (15)$$

where θ_i is the intersection angle between the wave vectors of incident light and ultrasound. Such diffraction is called Raman-Nath diffraction (with a small acouto-optic interaction length). Under Raman-Nath diffraction regime, the medium acts as a plane phase grating.

If the acousto-optic interaction length is larger than $(\lambda_s)^2/2\lambda_0$, it falls into the Bragg diffraction regime in which only $\pm 1^{st}$-order diffracted light occurs besides the undiffracted light (0^{th}-order) as shown in Figure 2. Here the medium acts as a volume grating and the incident angle is called the Bragg angle, satisfying the following condition:

$$\sin\theta_B = \frac{\lambda_0}{2\lambda_s} \quad (16)$$

Equation (16) is called the Bragg condition. As Bragg angle is quite small, the sum of incident angle and diffraction angle is

$$\varphi = 2\theta_B = \frac{\lambda_0}{v_s} f_s \quad (17)$$

where f_s and v_s are the frequency and velocity of ultrasound in the medium.

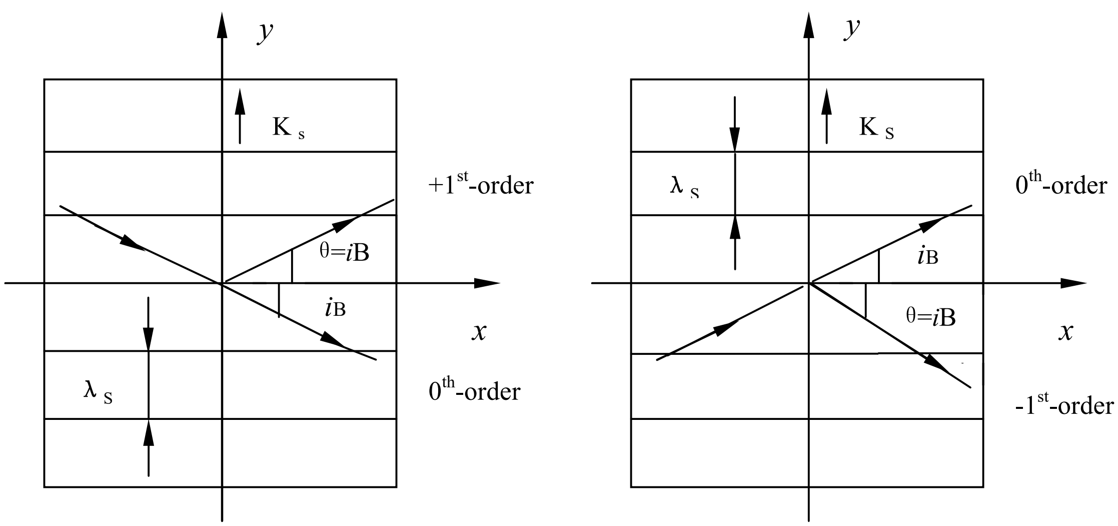

Figure 2 Schematic of Bragg diffraction

The efficiency of Bragg diffraction is determined by

$$\eta_B = \sin^2\left(\frac{\pi}{\lambda_0}\sqrt{\frac{M_2 L P_s}{2H}}\right) \quad (18)$$

where P_s is the power of ultrasound wave, L and H are the length and width of the medium, respectively, and M_2 is a constant of the acousto-optic medium.

$$M_2 = \frac{n^6 P^2}{\rho v_s^3} \tag{19}$$

where ρ is the medium density.

In theory, the efficiency of Bragg diffraction can reach 100% while the efficiency of the 1st-order diffracted light under Raman-Nath diffraction regime is only about 34%. Due to this fact, most acousto-optic devices are based on Bragg diffraction. According to equations (17) and (18), by altering the frequency and power of ultrasound, the direction and intensity of diffracted light can be controlled, respectively. So here comes acousto-optic deflector and acousto-optic modulator. It is known from equation (13) that the frequency of light diffracted by an acousto-optic device is also shifted, hence a frequency shifter can be made using acousto-optic effect. Such device has played an important role in laser Doppler anemometry.