2. Theory

2.1 Faraday Effect

1) As seen in Figure 1, if the applied magnetic field is not strong, then the rotation of the plane of polarization is proportional to the magnetic induction B and the length of the medium L,

$$\theta = VBL \text{ or } \theta = V\int_0^L Bdl \qquad (1)$$

where V is the Verdet constant, which depends on the medium and the wavelength of the light. It represents the magneto-optic characteristic of the medium.

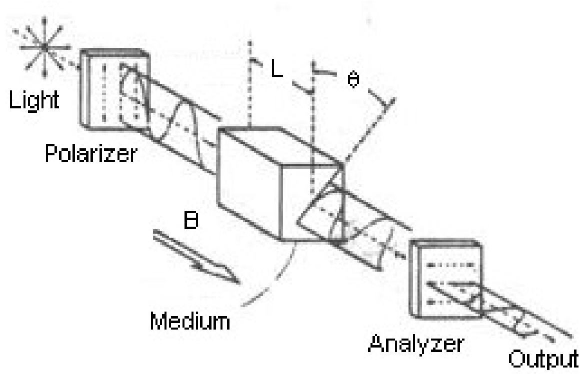

Figure 1 Schematic of Faraday effect

Almost all the media possess Faraday effect. However, the rotation direction of the plane of polarization may be different for different media. Assume the magnetic field is induced by a coil of metal wires wound round the medium, if the rotation of the plane of polarization is in the same direction as the current flow in the coil, then it is called positive rotation (V>0); otherwise, it is called negative rotation (V<0).

2) For a certain medium, the Faraday rotation direction is dependent solely on the direction of the magnetic field, but is independent of the propagation direction of light. This is the major difference between Faraday effect and other effects of optical activity. For an optically active medium, the direction of its polarization rotation depends on the propagation direction of the light. When viewed along and against the propagation direction of the light, the directions of the polarization rotation are against each other. As a result, if light travels back and forth to complete a round trip in an optically active medium, the plane of polarization of the light restores to the original state and therefore the polarization of the light does not rotate.

   By contrast, if the applied magnetic field is fixed, when light travels back and forth in a magneto-optic medium for one round trip, the Faraday rotatory angle is doubled. Therefore, light can be allowed to travel in the magneto-optic medium for many round trips to enhance the Faraday effect.

3) Similar to optical activity, optical rotatory dispersion also exists in Faraday effect. That is the Verdet constant is a function of the wavelength of light. When a ray of white light with linear polarization passes through a magneto-optic material, the polarization rotation of violet light is larger than that of red light. It has been proven by experiment that the Verdet constants of

magneto-optic material decrease with an increase in the wavelength of light. The curve of optical rotatory dispersion (V-λ) is also called the Faraday rotation spectrum.

2.2 Faraday Rotation Angle

A ray of light with linear polarization can be decomposed into two components of equal amplitude, left-circularly and right-circularly polarized light. Because the refractive index of a magneto-optic material is different for circularly polarized light with opposite rotation direction, an optical path difference is formed when the two components of circularly polarized light pass through the material

$$\delta = \left(n_R - n_L\right)L \tag{2}$$

where n_R and n_L are the refractive indices of the material for right-circularly and left-circularly polarized light, respectively.

When the two components of circularly polarized light are composed after passing through the medium, a ray of light with linear polarization is restored but the plane of polarization of the exit light rotates an angle as compared to that of the incident light. This angle is called the Faraday rotation angle, as defined by

$$\alpha_F = \frac{\omega}{2c}(n_R - n_L)L \tag{3}$$

Where ω is the angular frequency of light, and c is the speed of light in vacuum.

2.3 Calculation of Faraday Rotation Angle

From quantum theory, the magnetic dipole moment of an electron can be expressed by

$$\vec{\mu} = -\frac{e}{2m}\vec{L} \tag{4}$$

where e is the electric charge of an electron, m is the mass of an electron, and $\vec{L}$ is the angular momentum of an electron. In a magnetic field, the potential of an electron possesses is

$$P = -\mu B = \frac{eB}{2m}L_z \tag{5}$$

where L_z is the axial component of the angular momentum of the electron. When a ray of light with linear polarization passes through a medium in a magnetic field, photons and electrons interact resulting in an increase in the angular momentum of the electron. Hence, electrons transit to a new orbit with potential increased by

$$\Delta P = \frac{eB}{2m}\Delta L_z = \pm\frac{eB}{2m}\hbar \tag{6}$$

When photons with left-circular polarization interact with electrons, then equation (6) becomes

$$\Delta P_L = \frac{eB}{2m}\hbar \tag{7}$$

While photons with right-circular polarization interact with electrons, equation (6) becomes

$$\Delta P_R = -\frac{eB}{2m}\hbar \tag{8}$$

Due to the dispersion of the medium, the refractive index of the medium in a magnetic field is a function of photo energy. Thus

$$n = n(\hbar\omega) \tag{9}$$

One can assume that the interaction between an electron and a left-circularly polarized photon with energy of $\hbar\omega$ in the presence of a magnetic field is equivalent to that of an electron and a left-circularly polarized photon with energy of $\hbar\omega - \Delta P_L$ in the absence of a magnetic field. So

$$n_L(\hbar\omega) = n(\hbar\omega - \Delta P_L) \tag{10}$$

Equation (10) can be rewrittenas

$$n_L(\omega) = n(\omega - \frac{\Delta P_L}{\hbar}) = n(\omega) - \frac{dn}{d\omega}\frac{\Delta P_L}{\hbar} = n(\omega) - \frac{eB}{2m}\frac{dn}{d\omega} \tag{11}$$

Similarly, for a right-circularly polarized photon, we get

$$n_R(\omega) = n(\omega - \frac{\Delta P_R}{\hbar}) = n(\omega) - \frac{dn}{d\omega}\frac{\Delta P_R}{\hbar} = n(\omega) + \frac{eB}{2m}\frac{dn}{d\omega} \tag{12}$$

If equations (11) and (12) are substituted into equation (3), then

$$\alpha_F = \frac{eBL}{2mc}\omega\frac{dn}{dw} = \frac{eBL}{2mc}\lambda\frac{dn}{d\lambda} \tag{13}$$

When equation (13) is compared to equation (1), the Verdet constant of the medium is

$$V(\lambda) = \frac{e}{2mc}\lambda\frac{dn}{d\lambda} \tag{14}$$

Faraday rotation angle can be calculated based on equation (13), which shows that the Faraday rotation angle of a medium is proportional to the length of the medium and the magnetic induction applied. It is dependent on the wavelength of the light and the dispersion characteristic of the medium.

2.4 Magneto-Optic Modulation

If an AC electric current is supplied to the exciting coil, then an AC magnetic field is applied to the medium forming a magneto-optic modulator (now the exciting coil is called the modulating coil). If the magneto-optic modulator is placed between a polarizer and an analyser, and no current is supplied to the coil, then the output intensity of a linearly polarized light beam going through the system is defined by the Malus's law, as

$$I = I_0 \cos^2(\alpha) \quad (15)$$

where α is the intersection angle between the axes of the polarizer and the analyser.

When an AC current is supplied to the coil as $i = i_0\sin(\omega t)$, then an AC magnetic field is created as $B = B_0\sin(\omega t)$. Correspondingly, the Faraday rotation angle is modulated as $\alpha_F = \alpha_{F0}\sin(\omega t)$. Now the output intensity of the light exiting from the analyzer becomes

$$I = I_0 \cos^2(\alpha + \alpha_F) = \frac{I_0}{2}\left[1 + \cos 2(\alpha + \alpha_F)\right] = \frac{I_0}{2}\left[1 + \cos 2(\alpha + \alpha_{F0}\sin \omega t)\right] \quad (16)$$

where α_{F0} is the range of modulated Faraday rotation angle. Equation (16) describes the fundamental principle of magneto-optic modulation.

2.5 Basic Parameters of Magneto-Optic Modulation

There are two basic parameters to describe magneto-optic modulation, the modulation depth and the range of modulated angle.

- Modulation depth η

The modulation depth of a magneto-optic modulator is defined as

$$\eta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \quad (17)$$

where $I_{\max}$ and $I_{\min}$ are the maximum and minimum output intensities of the modulated light.

It is apparent from equation (16) that, if $0 \le (\alpha+\alpha_F) \le \pi/2$, then the maximum and minimum output intensities of the modulated light are

$$I_{\max} = \frac{I_0}{2}\left[1 + \cos 2(\alpha - \alpha_{F0})\right] \quad (18)$$

$$I_{\max} = \frac{I_0}{2}\left[1 + \cos 2(\alpha + \alpha_{F0})\right] \quad (19)$$

If the modulation amplitude of the optical intensity is $I_A = I_{\max} - I_{\min}$, then

$$I_A = I_0 \sin(2\alpha)\sin(2\alpha_{F0}) \quad (20)$$

Obviously, when the intersection angle between the axes of the polarizer and the analyzer is 45°, the modulation amplitude becomes maximum as

$$I_{A\max} = I_0 \sin(2\alpha_{F0}) \quad (21)$$

Now the modulation depth (α=45°) is

$$\eta = \sin(2\alpha_{F0}) \tag{22}$$

The range of modulated Faraday rotation angle (α=45°) can be calculated by

$$\alpha_{F0} = \frac{1}{2}\arcsin\left(\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}\right) \tag{23}$$