(3)Light source and light transmitting and receiving optics

A GaAs laser diode with a central wavelength of 650 nm is used as the light source whose output intensity is modulated sinusoidally at 100 MHz generated by a main oscillator. A schematic diagram of the light transmitting and receiving optics is shown in Figure 3.

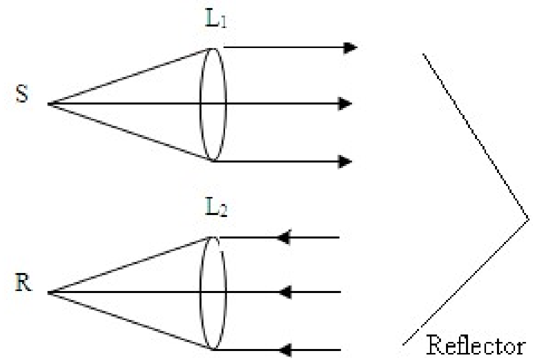

Figure 3 Light transmitting and receiving optics

(4) Light receiving system

A Silicon photodiode is used as the light receiving device. The phase of the measured signal has a delay, $\varphi = \varpi t$ (t is the time interval of light traveling the distance between transmitter and receiver), relative to the reference signal.

3. Theory

A. Wavelength measurement of a modulated wave using phase method

The speed of a wave is presented as $c=\lambda f$, where c, λ and f are the speed, wavelength and frequency of the wave, respectively. By measuring any two parameters, the third parameter can be determined.

However, for a light wave in the visible band, its frequency is very high at 10^{14} Hz and wavelength is very short at sub-microns. It is difficult to directly measure two parameters to determine the third. Instead, by using a wave with a relatively low frequency, such as 100 MHz as in this apparatus, to modulate the light wave, the speed of light can be determined with ease since the modulated light wave still propagates at the speed of the original light wave.

The intensity of a light wave modulated by a sinusoidal wave at frequency f can be expressed as:

$$I = I_0\left[1 + m\cos 2\pi f\left(t - \frac{x}{c}\right)\right] \quad (1)$$

where m is modulation rate, and light propagates along x axis. The phase of the modulation wave varies at a period of 2π during its propagation. If we can determine two locations on x axis of equal phase for the modulation wave, the distance between the two locations on x axis must be an integer multiplication of wavelength λ.

As shown in Figure 3, if the light wave at transmitter S is the reference signal while the reflected light wave at receiver R is the measurement signal, the phase difference between the two signals can be directly measured with a phase meter (oscilloscope). If the path length of the reflected signal changes by one wavelength, the phase difference will change by 2π. By moving the reflector back or forth along the rail until two adjacent locations of the reflector are identified with equal phase, the distance between the two locations is one half of a wavelength. As long as the frequency of the modulated wave is known, the speed of light can be determined.

B. Phase measurement of a modulated wave using frequency differential method

In practice, it is difficult to directly measure the phase of a high frequency signal such as 100 MHz at high accuracy. Instead, a phase differential method is usually used to transfer the measurement from high frequency signal to medium or low frequency signal.

When two sinusoidal signals of different frequency are input to a nonlinear element such as a diode or a transistor, the output signal contains a component at the frequency differential of the two input signals. In general, the output signal of a nonlinear element to an input signal x can be expressed as:

$$y(x)=A_0+A_1x+A_2x^2+\cdots \tag{2}$$

By ignoring the higher-order terms, the second order term generates a frequency mixing effect. If the high frequency reference and measurement signal are denoted as u_1 and u_2, respectively, as:

$$u_1 = U_{10}\cos(\omega t+\varphi_0) \tag{3}$$

$$u_2 = U_{20}\cos(\omega t+\varphi_0+\varphi) \tag{4}$$

By introducing a local oscillator, we get:

$$u' = U_0'\cos(\omega' t+\varphi_0') \tag{5}$$

where φ_0 and φ_0' are the initial phase of the reference and local oscillator signals, respectively; φ is the phase shift amount of the modulated light wave traveling through the optical path on the rail. By substituting (4) and (5) into (2) while omitting higher-order terms, we have:

$$y(u_2+u') \approx A_0 + A_1u_2 + A_1u' + A_2u_2^2 + A_2u'^2 + 2A_2u_2u' \tag{6}$$

By expending the cross term, we get:

$$\begin{aligned} 2A_2u_2u' &\approx 2A_2U_{20}U_0'\cos(\omega t+\varphi_0+\varphi)\cos(\omega' t+\varphi_0') \\ &= A_2U_{20}U_0'\{\cos[(\omega+\omega')t+(\varphi_0+\varphi_0')+\varphi]+\cos[(\omega-\omega')t+(\varphi_0-\varphi_0')+\varphi]\} \end{aligned} \tag{7}$$

Apart from the components of the two original frequencies, higher-order harmonic frequencies, and summing frequency, there is a component with the differential frequency of the two original signals as

$$A_2U_{20}U_0'\cos[(\omega-\omega')t+(\varphi_0-\varphi_0')+\varphi] \tag{8}$$

Similarly, the differential term of reference signal and local oscillator is:

$$A_2U_{10}U_0'\cos[(\omega-\omega')t+(\varphi_0-\varphi_0')] \tag{9}$$

Comparing (8) and (9), the two differential signals still have a phase difference of φ.

This experimental apparatus employs frequency differential and phase discrimination method by mixing high frequency (100 MHz) reference and measurement signals with a local oscillator at 100.455 MHz, respectively. As a result, two low frequency beat signals of 455 kHz with a phase difference of φ are generated. Figure 4 shows the block diagram of the experimental setup, in which Freq. Mixer I generates the low frequency (455 kHz) between reference signal and local oscillator while Freq. Mixer II generates the low frequency (455 kHz) between measurement signal and local oscillator. The amplitude of the low frequency measurement signal can be measured by using a millivolt meter.

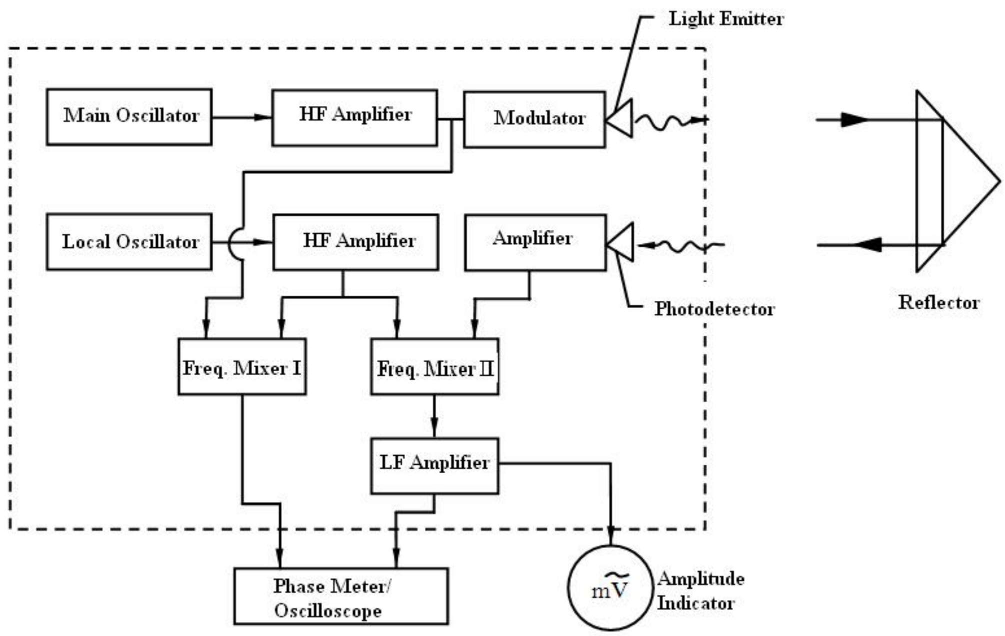

Figure 4 Schematic of frequency conversion and measurement

C. Phase measurement methods

(1) Using a digital phase meter (supplied by user)

Input the reference signal (port #4) and the measurement signal (port #5 or #6) into the two input ports of a phase meter. The two signals are shaped into square waves by bi-stable circuits in the meter. Then the phase difference between the two signals is determined by electronic circuits.

(2) Using an oscilloscope (supplied by user)

a) If a single-channel oscilloscope is used, set trigger synchronization mode as “External” and polarity as “+” or “-”.

Connect the reference signal (port #4) to the terminal of “External trigger synchronization input” and the measurement signal (port #5 or #6) to the terminal of “Y input”.

Adjust “Trigger Level” to achieve a stable waveform. Adjust the gain of the “Y axis” to set the amplitude of the waveform at proper level while adjusting time scale of horizontal axis to achieve a full waveform.

Count the divisions covered by a full waveform, for example, 10 divisions for 360° and then one division is 36°. Approximately, 0.1 division can be estimated, i.e. 3.6°.

First, record the position of a specified point of the waveform, then move the reflector carrier forth or back direction while observing the shift of the waveform on the oscilloscope and recording the position of the reflector at a specified point. Finally, use the displacement of the waveform on the oscilloscope to determine phase change as shown in Figure 5 as an example, here $\Delta\varphi = \frac{r}{r_0} \times 360^{\circ}$ or $\frac{r}{r_0} \times 2\pi$ in radians.

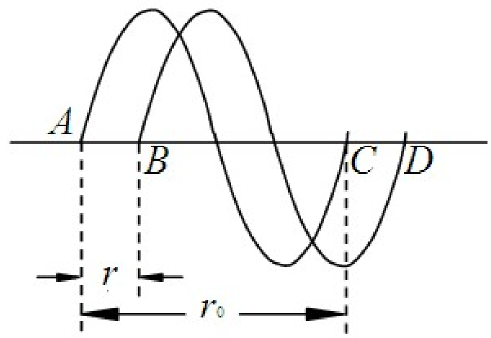

Figure 5 Schematic of waveform shift due to phase change

b) If a dual-channel oscilloscope is used, connect the reference signal (port #4) to Y1 channel and the measurement signal (port #5 or #6) to Y2 channel, use Y1 channel to trigger scan, and set display mode at "Continuous". Similar to the procedure described above, i.e. adjust gain and time scale, to achieve a full waveform and determine phase difference of the signal waveforms.

c) If a digital oscilloscope is used, use the calliper function of the cursor to accurately locate the two positions of the waveform before and after moving the reflector. **Ideally, one can acquire the waveform data from the digital oscilloscope and then conduct sinusoidal curve-fitting to the data so that the phase of a waveform can be accurately determined.**