3. Theory

3.1 Interference

Light is an electromagnetic wave associated with electric and magnetic fields. When two or more light waves overlap, the total light wave at any point and at any instant is governed by the law of superposition. As a result, the resultant electric or magnetic field at any point and at any instant is the addition of the instantaneous electric or magnetic fields produced at the point by the individual light waves. If the individual light waves have phases that bear no fixed relationship to each other over time, then the strength of the added electric or magnetic field at a point would vary randomly over time. Such strength, if averaged over time, would become more or less identical at all points on an observation screen. Under such circumstances, the screen would be more or less uniformly illuminated and an interference pattern would not be seen on the screen. These sources are called as incoherent sources.

By contrast, coherent sources are those whose output waves maintain a constant phase relation to each other over time. Usually, these light waves come from the same source so that they bear some degree of frequency and phase correlation between them. When the light waves from two coherent sources arrive at a point in phase, the field of the resultant wave is the sum of those of the individual waves; thus the individual waves reinforce each other, known as constructive interference. When the two coherent waves arrive at another point out of phase, the field of the resultant wave is the difference of those of the individual waves; hence the individual waves undermine each other, named as destructive interference. Thus, an interference phenomenon is observed only when the sources are coherent.

Light wave interference from two sources was first demonstrated by Thomas Young in 1801. Young designed an apparatus to allow a plane light wave to fall on two closely spaced parallel slits, serving as a pair of coherent light sources as waves emerging from them originate from the same wave front and therefore maintain a fixed phase relationship. The light from these two slits produces a visible pattern of bright and dark parallel bands called fringes on a viewing screen. Young's experiment obtained convincing evidence for the wave nature of light.

3.2 Michelson Interferometer

An important instrument involving wave interference is the Michelson interferometer invented by A. A. Michelson in 1881 using a similar principle. Now, Michelson interferometers have been used to measure wavelengths or other lengths with great precision.

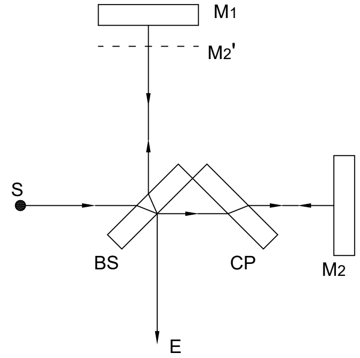

Figure 1 Diagram of Michelson interferometer

Figure 1 shows the schematic diagram of a Michelson interferometer. A ray of light from a monochromatic source **S** is split equally into two rays by a beam-splitter **BS**, which is inclined at 45° to the incident light beam. One beam is reflected by **BS** vertically upward toward a fixed mirror **M$_1$**, the second ray is transmitted horizontally through **BS** toward a movable mirror **M$_2$**. After reflecting from **M$_1$** and **M$_2$**, the two rays eventually recombine at **BS** to produce an interference pattern, which can be viewed by an observer's eye **E**. The purpose of using a compensator plate **CP** here is to ensure that the two rays pass through the same thickness of glass, as **CP** is cut from the same piece of glass as **BS** so that their thicknesses are identical.

The interference condition for the two rays is determined by their path differences. In general, the interference pattern is a target pattern of bright and dark circular fringes. As **M$_2$** is moved, the fringe pattern collapses or expands, depending upon the moving direction of **M$_2$**. In either case, the fringe pattern shifts by one-half fringe each time **M$_2$** is moved a distance that is equal to a quarter of the wavelength of light. As a result, the wavelength of light can be measured by counting the number of fringe shifts for a given displacement of **M$_2$**. On the other hand, if the wavelength of light is known, mirror displacement can be measured precisely, within a fraction of the wavelength of light using the same procedure.

3.3 Fabry-Perot Interferometer

When one beam of light passes through a plane-parallel plate with two reflecting surfaces, it is reflected many times between the two surfaces and hence multiple-beam interference occurs. The higher the surface reflectance is, the sharper the interference fringes are. That is the basic principle of a Fabry-Perot interferometer.

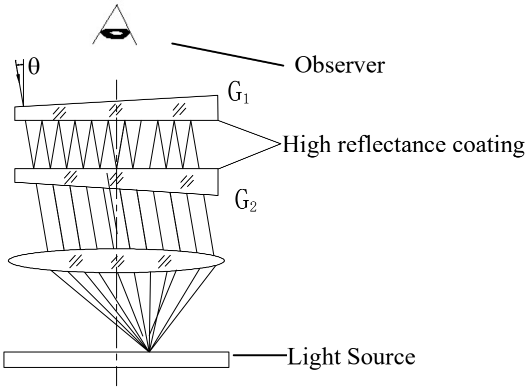

Figure 2 Schematic of Fabry-Perot interferometer

As shown in Figure 2, two partially reflecting mirrors **G$_1$** and **G$_2$** are aligned parallel to each other, which form a reflective cavity. When monochromatic light is incident on the reflective cavity with an angle θ, many parallel rays pass through the cavity to get transmitted. The optical path difference between two neighboring rays is given by δ:

$$\delta = 2nd\cos\theta$$

Thus, the transmitted light intensity is:

$$I' = I_0 \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\pi\delta}{\lambda}}$$

where I_0 is the incident light intensity, R is the mirror reflectivity, n is the refractive index of the medium in the cavity, d is the cavity length or mirror spacing, and λ is the wavelength of the monochromatic light.

Thus, I' varies with δ. When

$$\delta = m\lambda \qquad (m = 0, 1, 2\ldots)$$

I' becomes a maximum so that constructive interference of the transmitted light occurs; when

$$\delta = (2m' + 1)\lambda/2 \qquad (m' = 0, 1, 2\ldots)$$

I' becomes a minimum and destructive interference of the transmitted light is observed.

3.4 Twyman-Green Interferometer

Twyman-Green interferometer is a variant of the Michelson interferometer and it is mainly used to measure the defects in optical components such as lenses, prisms, windows, laser rods, and plane mirrors. Although the beam splitter and mirror arrangement in a Twyman-Green interferometer resembles a Michelson interferometer, there is a difference between these two interferometers. That is the light source used in a Michelson interferometer is usually an extended source (though it can also be a laser), while a Twyman-Green interferometer always uses a point light source such as a laser. The quality of an optical component under test can be evaluated from the irregularities of the interference pattern caused by placing the component into one beam path of a Twyman-Green interferometer. In particular, spherical aberration, coma, and astigmatism can be identified as specific variations in the fringe pattern.

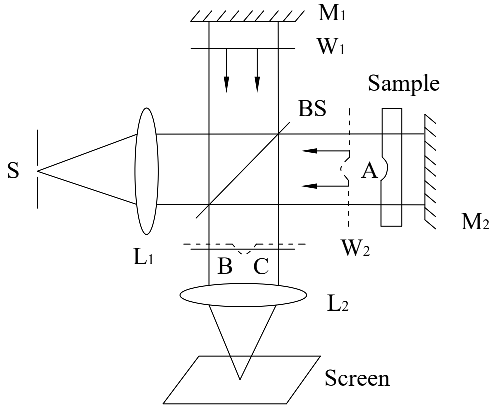

Figure 3 Schematic of Twyman-Green interferometer

In above figure, if the sample under test has perfectly flat surfaces, then the returning wave front is plane and no fringes are observed. However, if the optical flat is not perfectly flat on either side, the wave from M2 returning to the beam splitter is no longer plane. Thus, the phase difference between the superimposed waves of M1 and M2 will vary across the field of view and a fringe pattern will appear. These fringes form a contour map of the distorted wave front, so that the imperfections of the sample are displayed in terms of wave front aberrations.