4. Experiment contents

1) Measure gravity acceleration *g* using two methods:
   a. measure the height difference between the highest and the lowest points of the surface of rotating liquid, then calculate gravity acceleration g;
   b. laser beam incident parallel to the rotation axis to measure surface slope, then calculate gravity acceleration g;
2) Verify the relationship between focal length f and rotational period T according to the parabolic equation;
3) Study concave mirror imaging of rotating liquid surface.

5. Precautions

1) Turn the "Rotating Speed" adjustment knob to the minimum prior to turning on power;
2) Mount the transparent frame (with red reference lines) and the horizontal beam (i.e. horizontal ruler) onto the vertical post (i.e. vertical ruler). Mount the observation screen and the laser mount onto the horizontal beam.
3) Do not fill too much water into the cylindrical container to avoid overflow during rotation;
4) Before the rotary stage starts to rotate, please check its surroundings to make sure it will not hit the cables or other things;
5) Every time for measuring rotation period, press the "Timing" button, the counter is automatically cleared and the time of a single period is recorded. The recorded time can be saved in 10 groups (0-9). Press "Check" button to check the recorded time. The display brightness becomes dim after 10 times of counting, please press "Reset" button to restart counting.

6. Experiment procedure

1) Place the bubble level in the center of the rotation stage and adjust feet to level the stage.
2) Measure the inner diameter *D* of the cylindrical container with a vernier caliper and get radius $R = \frac{D}{2}$. Fill the cylindrical container with water near the mark line, place it on the stage, adjust the axis of the cylindrical container to be consistent with the rotation axis of the stage as close as possible, and then tighten the three screws on the side to fix the position of the container. Use the vertical ruler and the transparent frame to measure the height h_0 of liquid surface at rest state. Parallax can be eliminated by aligning the front long line with the rear short line on the transparent frame when observing the liquid surface from the viewing direction shown in Figure 5.

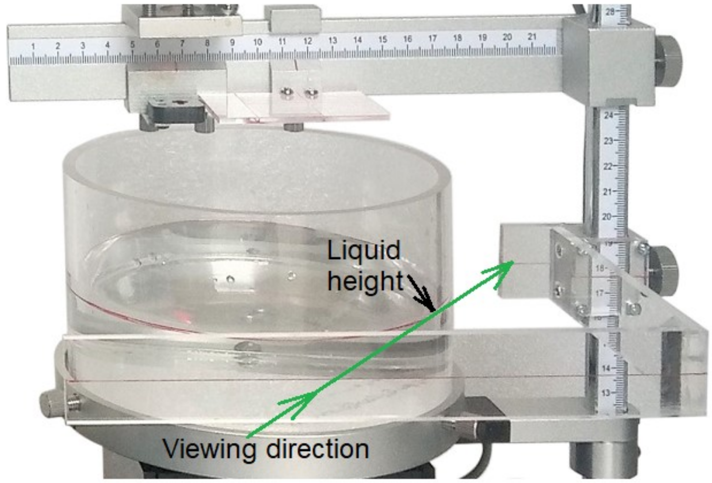

Figure 5 Schematic of eliminating parallax

3) Connect the cables of "Timer" and "Motor" between the platform and the main electric unit. Turn the "Rotating Speed" adjustment knob to the minimum, and turn on the power.

4) Measure gravity acceleration using the height difference between the highest and lowest points of the rotating liquid surface:

   Adjust the motor speed, measure the rotation period T of the stage. Use the vertical ruler to measure the height h of the highest point of the liquid surface and the height y_0 of the lowest point, then $\Delta h = h - y_0$. Use equation (11) to calculate the acceleration of gravity g. Note: to ensure the accuracy of Δh, the speed cannot be too slow. Change the rotation period T to do multiple measurements to get multiple sets of h and y_0, obtain multiple g values, and the average value $\bar{g}$ is obtained with improved measurement accuracy.

5) Measure the gravity acceleration by slope measurement method using a parallel laser beam:

   a) Turn the "Rotating Speed" adjustment knob to the minimum. Mount the spot beam laser (the shorter one) onto the 2-D adjustment laser support on the horizontal beam, and connect it to the "Laser" (i.e. laser power supply) port on rear panel of the main electric unit. Note: the focus of the laser is adjustable. Adjust the laser head to achieve parallel beam prior to mounting it on the support.

   b) Adjust the horizontal beam (i.e. the horizontal ruler) to a certain height (e.g. around 35 cm), move the laser to make the laser beam to illuminate onto the water surface (e.g. locate the laser at round 5 cm scale line on the horizontal ruler), observe the reflected light spot, finely adjust the angle of the laser so that the beam is reflected by the water surface and returns to the original path, that is the reflected laser point returns to the exit of the laser head (may also use the transparent portion of the observation screen to pick up the reflected point). At this time, both the incident light and the reflected light are perpendicular to the water surface and parallel to the rotation axis;

   c) Adjust the motor speed to rotate the rotation stage. When the liquid surface is stable, let the reflected light spot shift 8 to 12 cm horizontally on the plane at the beam height. Finely rotate the horizontal beam around the vertical post, move the observation screen (i.e. the plastic plate with red grid lines) along the horizontal ruler to capture the light spot and locate the light spot on the middle line of the three parallel lines on the observation screen. At this time, the horizontal projection of the light beam is located on the diameter of the cylindrical container. Lock the beam and record the height *H*;

d) Reduce the rotation speed to zero again, and shift the laser along the horizontal beam to let the light spot fall on the circular engraving line on the stage, i.e. at $x_0 = \frac{R}{\sqrt{2}}$. Record the position of the laser on the beam as x_0;

e) Adjust the speed of the motor, and measure the position x of the reflected light spot on the observation screen under different rotation periods T;

f) Calculate $tan(2\theta)$ using x_0, x, H and h_0, and use formula $\tan\theta = \frac{\sqrt{1+\tan^2 2\theta}-1}{\tan 2\theta}$ to obtain $tan(\theta)$. Plot $\tan\theta - \frac{1}{T^2}$ graph using multiple sets of measurement results. Do linear fitting using the least square method to obtain the slope k. Finally, calculate the acceleration of gravity g using equation (13).

6) Verify the relationship between focal length f and rotational period T based on the parabolic equation

a) Adjust the motor speed to rotate the stage. When the liquid surface is stable, move the laser to find an incident point where the incident light beam and the reflected light beam coincide, this is the center of the liquid paraboloid. At this time, the incident and reflected beams coincide with the rotation axis. Lock the beam, and record the position of the laser on the beam as x_r;

b) Horizontally shift the laser while keeping the incident beam parallel to the rotation axis, let the laser beam deviate from the rotation axis, and move the observation screen to the position x_r of the rotation axis and lock it;

c) Adjust the speed of the motor and then move the horizontal beam up and down, make the reflected light spot locate to the middle point of the three intersection points of the four grid lines on the observation screen. At this time, the height difference between the horizontal beam and the lowest point of the liquid surface is exactly equal to the focal length of the parabola. Record the horizontal beam height *H*, the height of the lowest point of the liquid surface y_0 and the rotation period *T*. Note that when the observation screen is located in the focal plane of the concave parabolic mirror, the reflected laser point on the observation screen (also on the axis) will not move even the laser head moves horizontally.

d) Do multiple measurements and record multiple sets of data, use $f = H - y_0$ and plot $\ln f - \ln T$ relationship diagram. Use the least squares method to do linear fitting. According to the slope and intercept, calculate a and b in $\ln f = b\ln T + \ln a$. Compare them with the theoretical formula (14). Verify whether the rotating liquid surface obeys the parabolic law.

7) Study concave mirror imaging of rotating liquid surface

a) Replace the point laser with the divergent laser (the longer one), and turn the arrow plate below the laser head to the laser beam exit position. Before fixing the laser tube, rotate the tube to make the long side of the laser pattern along the arrow direction so that the arrow can be illuminated completely. The arrow is used as the object in the concave mirror imaging system. Let the arrow pattern completely projected onto the liquid surface.

b) Adjust the horizontal beam to a certain height *H*, move the observation screen to capture the image of the arrow reflected by the liquid surface. Change the rotation period *T* and observe the change in the image size and direction of the arrow. When the length of the image is equal to the length of the object (Note that the distance between the front line and back line of the three horizontal lines on the observation screen is exactly equal to the length of the arrow of the object), record the height y_0 of the lowest point of the liquid surface;

c) Change the height *H* of the beam and repeat step b);

d) Record the observed phenomenon in a table. Since the arrow plate and the observation screen are always at the same height, when the real image is same size as the object but in opposite directions, the radius of curvature of the liquid surface can be calculated as $r = H - y_0$. Then focal length is $f' = \frac{r}{2}$. Compare it with the theoretical value *f* calculated by equation (14).

7. Examples of Data Recording and Processing

Note: Following data are for reference purpose only, not the criteria for apparatus performance:

The inner diameter of the cylindrical container D = 141.5mm, the radius R = D/2 = 70.8 mm, and the liquid surface height h_0 = 176.0 mm at static state.

1) Measure gravity acceleration using the height difference between the highest and lowest points of the rotating liquid surface

Table 2 Data of height difference Δh at different rotation periods *T* and the calculated gravity acceleration g (note: T was measured 5 times at each position of the adjust knob of motor speed).

T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)	T_5 (s)	$\overline{T}$ (s)	h (mm)	y_0 (mm)	Δh (mm)	g (m/s^2)	$\overline{g}$ (m/s^2)
0.548	0.550	0.551	0.544	0.551	0.549	191.2	159.2	32.0	10.27	
0.595	0.595	0.595	0.594	0.592	0.594	190.1	160.7	29.4	9.53	
0.647	0.645	0.641	0.647	0.653	0.647	187.4	162.6	24.8	9.54	9.53
0.700	0.703	0.701	0.701	0.705	0.702	186.4	164.7	21.7	9.25	
0.745	0.751	0.746	0.745	0.745	0.746	185.6	166.0	19.6	9.06	

The average value of gravity acceleration $\overline{g} = 9.53 m/s^2$ is obtained. The accepted value of gravity acceleration is 9.8 m/s^2 (please use your local value), error is 2.7%.

2) Measure the gravity acceleration by slope measurement method using a parallel laser beam

Horizontal beam height *H* = 400 mm, laser position at $\frac{R}{\sqrt{2}}$ is x_0 = 44.8 mm.

Table 3 Reflected spot position *d* and slope *tan(θ)* under different rotation period *T*

T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)	T_5 (s)	$\overline{T}$ (s)	$\frac{1}{\overline{T}^2}$ (s^{-2})	x (mm)	d (mm)	$\tan 2\theta$	$\tan\theta$
1.621	1.624	1.621	1.619	1.619	1.621	0.381	78.6	33.8	0.1509	0.0750
1.526	1.524	1.519	1.526	1.520	1.523	0.431	83.1	38.3	0.1710	0.0849

1.420	1.421	1.421	1.419	1.417	1.420	0.496	89.2	44.4	0.1982	0.0982
1.330	1.327	1.331	1.324	1.326	1.328	0.567	96.8	52.0	0.2321	0.1145
1.221	1.220	1.219	1.219	1.219	1.220	0.672	105.5	60.7	0.2710	0.1331
1.123	1.129	1.128	1.126	1.125	1.126	0.788	116.1	71.3	0.3183	0.1553
0.999	1.001	1.000	0.997	0.999	0.999	1.002	136.7	91.9	0.4103	0.1972
0.949	0.950	0.946	0.950	0.954	0.950	1.108	146.2	101.4	0.4527	0.2158
0.900	0.903	0.903	0.901	0.905	0.902	1.228	160.1	115.3	0.5147	0.2423
0.851	0.848	0.842	0.841	0.844	0.845	1.400	177.5	132.7	0.5924	0.2740
0.800	0.804	0.801	0.803	0.799	0.801	1.557	196.6	151.8	0.6777	0.3069

Plot the $\tan\theta - \frac{1}{T^2}$ relationship graph, and use the least square method to do linear fitting, results are shown in Figure 6.

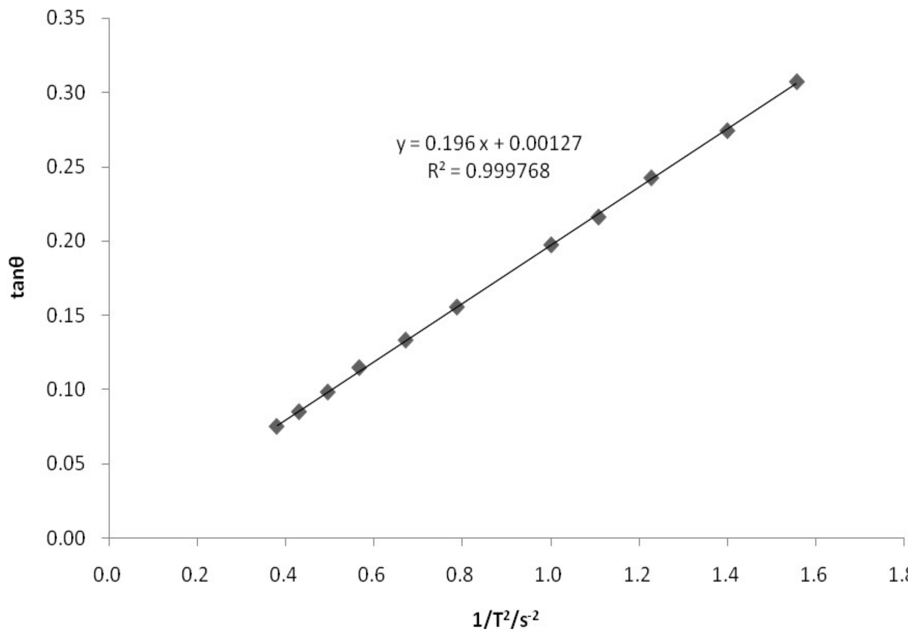

Figure 6 Linear fitting graph of slope *tan(θ)* and reciprocal square of rotation period $1/T^2$

We got slope k = 0.196, correlation coefficient R = 0.99988, and the acceleration of gravity $g = \frac{2\sqrt{2}\pi^2 R}{k} = 10.1\text{m/s}^2$. Experiment error is about 3.0%.

3) Verify the relationship between focal length f and rotational period T according to the parabolic equation

The rotation axis position is x_r = 94.9 mm.

Table 3 Focal length f under different rotation period T

T_1 (s)	T_2 (s)	T_3 (s)	T_4 (s)	T_5 (s)	$\overline{T}$ (s)	$\ln\overline{T}$	H (mm)	y_0 (mm)	f (mm)	f (m)	lnf
0.841	0.847	0.845	0.845	0.845	0.845	-0.1689	257.0	167.8	89.2	0.0892	-2.417
0.964	0.961	0.956	0.958	0.947	0.957	-0.0437	282.2	170.1	112.1	0.1121	-2.188
1.031	1.039	1.035	1.029	1.030	1.033	0.0323	302.0	170.8	131.2	0.1312	-2.031
1.105	1.108	1.109	1.107	1.111	1.108	0.1026	319.2	171.0	148.2	0.1482	-1.909
1.146	1.166	1.163	1.159	1.151	1.157	0.1458	334.8	171.1	163.7	0.1637	-1.810

1.187	1.188	1.192	1.180	1.180	1.185	0.1701	347.5	171.5	176.0	0.1760	-1.737
1.229	1.246	1.235	1.223	1.238	1.234	0.2104	360.0	171.6	188.4	0.1884	-1.669
1.276	1.272	1.276	1.284	1.277	1.277	0.2445	376.0	171.7	204.3	0.2043	-1.588
1.321	1.319	1.320	1.314	1.319	1.319	0.2766	387.3	171.8	215.5	0.2155	-1.535
1.392	1.411	1.394	1.400	1.412	1.402	0.3378	409.2	172.0	237.2	0.2372	-1.439
1.496	1.513	1.519	1.528	1.497	1.511	0.4125	444.0	172.2	271.8	0.2718	-1.303

Plot the $\ln f - \ln T$ relationship graph, use the least square method to do linear fitting, we got results shown in Figure 7.

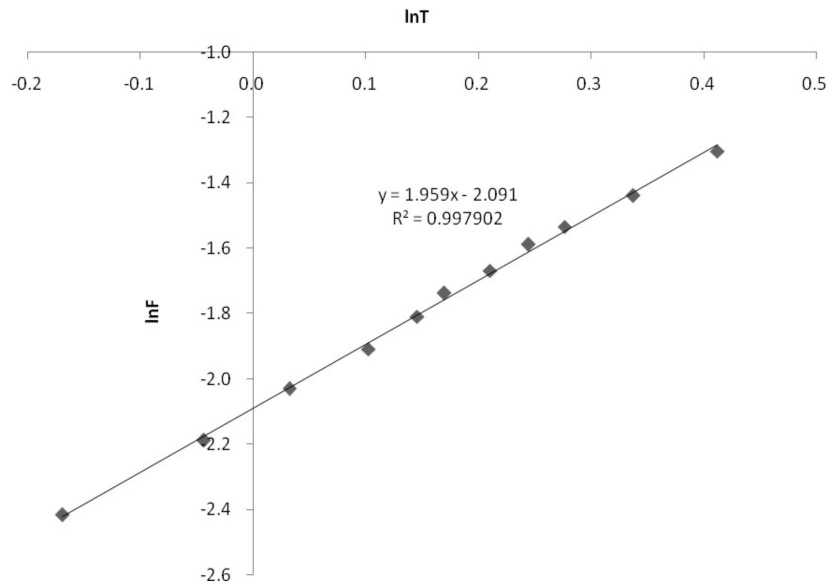

Figure 7 Linear fitted graph of *lnf* and *lnT*

We got slope b = 1.959, intercept lna = -2.091, i.e. $a = e^{-2.091} = 0.1236$, correlation coefficient R = 0.99895, then the relationship between focal length *f* and rotation period *T* is $f = aT^b = 0.1236T^{1.959}$. The theoretical formula is $f = \frac{g}{8\pi^2}T^2 = 0.1240T^2$. It can be seen that the measurement result is quite close to the theoretical formula.

4) Study concave mirror imaging of rotating liquid surface

Table 4 Imaging states under different rotation periods *T* for beam height *H* = 400.0 mm

Period	Image size	Image direction	Real/Virtual
T > 1.149 s	Enlarged	Erected	Virtual
T = 1.149 s	Equal	Erected	Virtual
1.027 s < T < 1.149 s	Reduced	Erected	Virtual
T = 1.027 s	A spot		
0.948 s < T < 1.027 s	Reduced	Reversed	Real
T = 0.948 s	Equal	Reversed	Real

T < 0.948 s	Enlarged	Reversed	Real

When T = 0.948 s, the height of the lowest point of the liquid surface is $y_0 = 171.1\text{mm}$, the radius of curvature $r = H - y_0 = 229.9\text{mm}$, then the measured value of the focal length $f' = \frac{r}{2} = 115.0\text{mm}$. At this time, the theoretical value of the focal length $f = \frac{g}{8\pi^2}T^2 = 0.1115\text{m} = 111.5\text{mm}$. Experiment error is about 3.1%.

Table 5 Imaging states under different rotation periods *T* for beam height *H* = 300.0 mm

Period	Image size	Image direction	Real/Virtual
T > 0.980 s	Enlarged	Erected	Virtual
T = 0.980 s	Equal	Erected	Virtual
0.810 s < T < 0.980 s	Reduced	Erected	Virtual
T = 0.810 s	A spot		
0.723 s < T < 0.810 s	Reduced	Reversed	Real
T = 0.723 s	Equal	Reversed	Real
T < 0.723 s	Enlarged	Reversed	Real

When T = 0.723 s, the height of the lowest point of the liquid surface $y_0 = 167.5\text{mm}$, the radius of curvature $r = H - y_0 = 132.5\text{mm}$, then the measured value of the focal length $f' = \frac{r}{2} = 66.3\text{mm}$. At this time, the theoretical value of the focal length $f = \frac{g}{8\pi^2}T^2 = 0.0648\text{m} = 64.8\text{mm}$. Experiment error is about 2.3%.