## 2. Theory

Different liquid has different degree of viscosity. When liquid flows, the speed of different layers parallel to the flowing direction is not same, i.e. relative sliding occurs, so friction forces are generated among layers called viscous forces whose direction is parallel to the surface in contact and whose amount is proportional to speed gradient and contact area. The proportion coefficient is called viscosity, which is an important parameter of a liquid.

The measurement of liquid viscosity has significant applications in medical, industrial and research fields. There are several methods for the measurement of liquid viscosity. The method adopted in this experiment is a falling ball method which is an absolute measurement method. When a small ball falls vertically into a sticky liquid, it is impacted by the viscous resistance of the liquid due to the relative motion between the liquid layer attached to the surface of the ball and the surrounding layers. The amount of the resistance is related to falling speed. When the ball moves uniformly in the liquid, liquid viscosity can be calculated by measuring the speed of the ball in the liquid.

After falling into a liquid, a metal ball is subject to three forces in the vertical direction: gravity $mg$ ($m$ is the mass of the ball, $g$ is the gravitational constant), buoyancy $pgV$ ($V$ is the volume of the ball, $p$ is the density of the liquid), and viscous resistance $F$ whose direction is opposite to the moving direction of the ball. If the depth and width of the liquid are infinite, and the falling speed of the ball ν is relatively low, we get the so-called Stokes formula:

$$F = 6\pi\eta r v \tag{1}$$

where $r$ is the radius of the ball; $\eta$ is the viscosity of the liquid in unit of Pa·s.

The viscous resistance encountered by the falling ball in the liquid increases with an increase in the falling speed of the ball. Eventually, three forces are balanced, namely,

$$mg = \rho g v + 6\pi\eta r v$$

Now, the small ball undertakes a uniform rectilinear motion in the liquid, so we have:

$$\eta = \frac{(m - V\rho)g}{6\pi v r}$$

By substituting $m=\pi d^3 p'/6$, $v=l/t$, and $r=d/2$ into the above expression, we get:

$$\eta = \frac{(\rho' - \rho)gd^2 t}{18l} \tag{2}$$

where $p'$ is the density of the ball material, $d$ is the diameter of the ball, $l$ is the distance of the ball falling in uniform speed, and $t$ is the time of the ball falling within distance $l$.

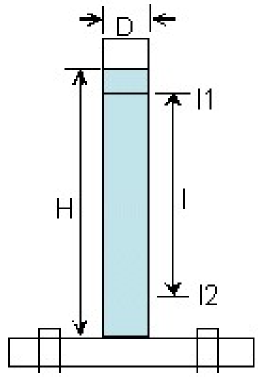


Figure 1 Schematic of experimental method

As shown in Figure 1, a liquid under test is stored in a container which does not have infinite depth and width. However, if a ball falls along the central axis of the barrel container, Eq. (2) can be corrected as follows:

$$\eta = \frac{(\rho' - \rho) g d^2 t}{18 l} \frac{1}{(1 + 2.4\frac{d}{D})(1 + 1.6\frac{d}{H})} \tag{3}$$

where $D$ is the internal diameter of the container, and $H$ is the height of the liquid.

If a metal ball falls fast in a liquid, such as a steel ball falls in hot oil, turbulence could occur. Under this case, formula (1) is no longer valid, which should be corrected (see Appendix 1).