3. Theory

A. Definition of Terms

1) Collision: refers to the phenomenon that the movement state changes rapidly when two moving objects contact each other. There are two types of collisions between two bodies: a) Head-on collision, where the velocity of each body just before impact is along the line of impact, and b) Oblique collision, where the velocity of each body just before impact is not along the line of impact.

2) Momentum conservation of collision: the total momentum before and after the collision between the two objects remains unchanged.

3) Horizontal throw motion: throwing an object in horizontal direction at a certain initial speed v_0, without considering the air resistance, the motion of the object is called a horizontal throw motion, and the kinematics equation is $x = v_0 t$, $y = \frac{1}{2} g t^2$, in which *t* is the time starting from the throw occurring, *x* is the horizontal distance that the object moves in time *t*, *y* is the vertical distance that the object falls during the time *t*, and *g* is the acceleration of gravity.

4) In the gravitational field, when the object of mass *m* is lifted a height *h*, its potential energy increases $E_p = mgh$.

5) When an object of mass *m* is moving at a speed v, its kinetic energy is $E_k = \frac{1}{2} m v^2$.

6) Law of transformation and conservation of mechanical energy: In the process of mutual conversion of potential energy and kinetic energy in any object system, if the work done by the external force on the object system is zero and the internal force is conservative (no dissipative force), then the total mechanical energy of the object system (i.e. the sum of potential energy and kinetic energy) remains constant.

7) Elastic collision: collision without loss of mechanical energy.

8) Inelastic collision: the mechanical energy in the collision process is not conserved, and part of it is converted into non-mechanical energy (such as thermal energy).

B. Derivation of Formula

When two balls collide elastically, if the energy loss is not considered, we have:

$$\begin{cases} m_1 v_1 = m_1 v_1' + m_2 v_2' & \text{(conservation of momentum)} \\ \frac{1}{2} m_1 {v_1}^2 = \frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2 & \text{(conservation of kinetic energy)} \end{cases}$$

where m_1 and m_2 are respectively the masses of the swing ball and the collided ball; v_1, v_1' and v_2' are speeds of before and after collision of the two balls. Combining the two formulas, we can get:

$$\begin{cases} v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \\ v_2' = \frac{2m_1}{m_1 + m_2} v_1 \end{cases} \qquad (1)$$

If the flying horizontal distance of the collided ball is measured as x, we have:

$x = v_2' t$, then: $v_2' = \frac{x}{t}$. (2)

where t is the flying time of the collided ball after collision. The falling height y of the collided ball is:

$y = \frac{1}{2} g t^2$, then $t = \sqrt{\frac{2y}{g}}$. (3)

Substituting formula (3) into formula (2), we get:

$$v_2' = \frac{x}{\sqrt{\frac{2y}{g}}} \qquad (4)$$

According to the law of conservation of kinetic energy, it can be seen that the kinetic energy of the swing ball immediately before the impact is $\frac{1}{2} m_1 {v_1}^2 = m_1 g h$ and its speed before collision can be obtained as:

$$v_1 = \sqrt{2gh} \qquad (5)$$

Substituting (4) and (5) into (1), we get:

$$\frac{x}{\sqrt{\frac{2y}{g}}} = \frac{2m_1}{m_1 + m_2} \cdot \sqrt{2gh} \, . \qquad (6)$$

After squaring and rearranging both sides of (6), we get:

$$h = \frac{(m_1+m_2)^2 x^2}{16 m_1^2 y} \quad (7)$$

Then, in the case of ignoring energy loss, when the mass of the swing ball and the collided ball is equal, that is, $m_1=m_2$, the theoretical release height of the swing ball should be:

$$h = \frac{x^2}{4y} \quad (8)$$

However, during the experiment, the collision between the two balls is not an ideal elastic collision. Therefore, when the falling height of the swing ball is *h*, due to the energy loss during impact, the actual landing point of the collided ball $x' < x$, then the actual speed of the collided ball after collision will be

$$v_2'' = \frac{x'}{\sqrt{\frac{2y}{g}}}, \quad (9)$$

The energy lost during the inelastic collision can be obtained as:

$$\begin{aligned}\Delta E &= \frac{1}{2} m_2 {v_2'}^2 - \frac{1}{2} m_2 {v_2''}^2 \\ &= \frac{1}{2} m_2 \left(\frac{x}{\sqrt{\frac{2y}{g}}}\right)^2 - \frac{1}{2} m_2 \left(\frac{x'}{\sqrt{\frac{2y}{g}}}\right)^2 \\ &= \frac{1}{2} m_2 \left(\frac{x}{\sqrt{\frac{2y}{g}}}\right)^2 - \frac{1}{2} m_2 \left(\frac{x'}{\sqrt{\frac{2y}{g}}}\right)^2 \\ &= m_2 g \left(\frac{x^2 - x'^2}{4y}\right)\end{aligned} \quad (10)$$

To make the collided ball hit an expected target center position *x*, the height of the swing ball should be raised to h_1 such that:

$$m_1 g h_1 - m_1 g h = \Delta E = m_2 g \left(\frac{x^2 - x'^2}{4y}\right) \quad (11)$$

The height difference that needs to be raised for the swing ball is:

$$\Delta h = h_1 - h = \frac{m_2}{m_1} \cdot \frac{x^2 - x'^2}{4y} \quad (12)$$

When $m_1 = m_2$:

$$\Delta h = \frac{x^2 - x'^2}{4y} \quad . \qquad (13)$$

4. Experiment contents

1) Study the collision between two balls of same mass. Measure the energy loss of the collision.
2) Study the collision between two balls of different masses. Measure the energy loss of the collision.

5. Experiment procedure

1) Observe the motion of the swing ball when the electromagnet is turned off, and observe the movement states before and after the two balls collision. Measure the energy loss of the collision.

   a) Adjust the level of the guide rail by adjusting the two adjustment screws on the guide rail; check the levelness using a line level.

   b) Use an electronic balance to measure the mass of the collided ball (both the diameter and material are the same as the swing ball), and also use this mass as the mass of the swing ball.

   c) Set and measure the position x, i.e. the target point (or the center of the target plate), where it is expected the collided ball falls there after collision and measure the height y of the collided ball on the support rod before collision. According to the position x and height y, calculate the height h of the swing ball accordingly (the formula for calculating the height h from x and y should be derived by students in advance).

   d) Adjust the height, left and right of the swing ball finely so that it can make a front collision with the collided ball at the lowest point of the swing ball.

   e) Attract the swing ball under the electromagnet and adjust the height of the magnet on the "Scaled post 4" to let the swing ball achieve the expected height h. Then move the position of the scaled post along the rail to make the string straight.

   f) Let the swing ball collide the collided ball. Record the position where the collided ball hits the target plate in the tray. (this step can be repeated multiple times and do average)

Consider how should the height of the swing ball be adjusted to hit the preset position x exactly? (In advance the lab class, the formula between height adjustment and position x error should be derived, i.e. the formula between $\Delta h = h - h_0$ and x, Δx and y.)

g) After adjusting the height of the swing ball, repeat the collision several times to determine the best height value h that can hit the preset x exactly. Then calculate the total energy loss before and after collision.

h) Observe the movement state of the two small balls before and after collision, and analyze various causes for energy losses before and after collision.

2) Observe the movement state of two balls of different masses before and after collision, and measure the energy loss before and after collision.

Using the other collided ball of different diameter and mass, repeat the above experiment, compare the experimental results and discuss them. (Note: Due to different diameter, the height of the electromagnet and the string should be readjusted.)

Precautions

1) Avoid adjusting the bottom end of the armature screw too low, and ensure that the swing ball is in close contact with the armature. For repeating measurement, ensure the contact between the swing ball and the armature at the same position every time.

2) When the ball is swinging, the Swing Post must not shake, and the relevant fixing screws must be tightened.

3) When the swing ball is attracted to the electromagnet, the string should be in a straight line (must not have obvious slack).

6. Examples of Data Recording and Processing

Note: Following data are for reference purpose only, not the criteria for apparatus performance:

From the experimental theory, we got $h = \dfrac{(m_1+m_2)^2 x^2}{16 m_1^2 y}$;

when $m_1 = m_2$, it is $h = \dfrac{x^2}{4y}$; then $\Delta h = \dfrac{x^2 - x'^2}{4y}$.

Table 1 Parameters before collision ($m = m_1 = m_2$)

Ball mass m (g)	Ball diameter d (cm)	y (cm)	x (cm)	Calculated height h (cm)
32.57	2.00	15.5	21.8	7.7

Table 2 Recorded measurement data

h (cm)	Measurement No.	Falling position x' (cm)	Average $\overline{x'}$ (cm)	Correction amount Δh (cm)
7.7	1	19.5	19.5	1.5
	2	19.5		
	3	19.7		
	4	19.5		
	5	19.3		

1st correction h_1 (cm)	Measurement No.	Falling position x' (cm)	Average $\overline{x'}$ (cm)	Correction amount Δh (cm)
9.2	1	21.3	21.4	0.3
	2	21.4		
	3	21.4		
	4	21.4		
	5	21.5		

2nd correction h_2 (cm)	Measurement No.	Falling position x' (cm)	Average $\overline{x'}$ (cm)	Correction amount Δh (cm)
9.5	1	21.7	21.8	0
	2	21.8		
	3	21.8		
	4	21.9		
	5	21.7		

Conclusion: the h value that can exactly hit the target center is 9.5 cm. The local acceleration of gravity is $g = 9.794\ m/s^2$, so the total energy loss of the collision is:

$$\Delta E = mg(h_1 - h) = 32.57\times10^{-3}\times9.794\times(9.5\times10^{-2} - 7.7\times10^{-2}) = 5.74\times10^{-3}\,J$$

Find the class A uncertainty in collision and horizontal throw motion:

Use h = 7.7 *cm* and y = 15.5 *cm* as an example, the data of measured x is given in Table 3

Table 3 Under the same conditions, repeat the measurement 10 times

Measurement times	1	2	3	4	5	6	7	8	9	10
Position x	19.5	19.5	19.7	19.5	19.3	19.4	19.6	19.5	19.6	19.4

$\overline{x}=\frac{1}{n}\sum_{i=1}^{n} x_i=19.5cm$， $U_A=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}}=0.12cm$， so we have $x \pm U_A(x)=(19.5 \pm 0.12)cm$