3. Theory

1) Simple harmonic vibration and damped vibration

Many vibration systems, such as the vibration of a spring vibrator, the vibration of a single pendulum, and the vibration of a torsional pendulum, can be treated as simple harmonic vibrations when the amplitude is small and air damping can be ignored. That is, this type of vibration satisfies the simple harmonic motion equation

$$\frac{d^2x}{dt^2}+\omega_0^2 x=0. \quad (1)$$

The solution of (1) is

$$x=A\cos(\omega_0 t+\varphi). \quad (2)$$

For the vibration frequency of a spring oscillator $\omega_0=\sqrt{\frac{K}{m+m_0}}$, K is the spring stiffness, m is the mass of the oscillator, and m_0 is the equivalent mass of the spring. The period T of the spring oscillator satisfies:

$$T^2=\frac{4\pi^2}{K}(m+m_0). \quad (3)$$

However, there are various damping factors in the actual vibration system, so the damping term must be added to the left side of equation (1). In the case of small damping, the damping is proportional to the speed, expressed as $2\beta\frac{dx}{dt}$, then the corresponding damped vibration equation is:

$$\frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega_0^2 x=0, \quad (4)$$

where β is the damping coefficient.

2) Forced vibration and resonance

The amplitude of the damped vibration will decay with time, and finally the vibration will stop. In order for the vibration to continue, an external force of periodical changes must be given to the system. Generally a forced force of sine function or cosine function with time is used. Under the action of the forced force, the motion of the vibration system satisfies the following equation

$$\frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega_0^2 x=\frac{F}{m'}\cos\omega t, \quad (5)$$

where $m'=m+m_0$ is the mass of the vibration system, F is the amplitude of the forcing force, and ω is the circular frequency of the forcing force.

Equation (5) is the equation for the forced vibration of the vibration system. Its solution includes two items. One item is transient vibration. Due to the existence of damping, the amplitude of the vibration continues to decay after the vibration begins, and finally it becomes zero quickly. The other term is the solution for steady-state vibration, which is:

$x = A\cos(\omega t + \varphi)$, the amplitude $A = \dfrac{F/m'}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}$.

When the circular frequency of the forcing force $\omega = \omega_0$, the amplitude A will have a maximum value, which is called resonance at this time. Obviously, the smaller β, the greater the extreme value of the x ~ ω relationship curve. The physical quantity describing the steepness of the curve is sharpness, and its value is equal to the quality factor:

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1}.$$

3) Vibration period of adjustable frequency tuning fork

Once an adjustable frequency tuning fork starts to vibrate, it will vibrate at a fundamental frequency without vibrating at harmonic frequencies. The two arms of the tuning fork are symmetrical so that the vibrations of the two arms are completely reversed, so that at any instant there is an equivalent reverse force on the center rod. The net force of the center rod is zero without vibration, so holding it tightly will not cause vibration attenuation. For the same reason, the two arms of the tuning fork cannot move in the same direction, because the same direction movement will produce an oscillating force on the center rod, which will cause the vibration to attenuate quickly.

The fundamental frequency of the tuning fork can be reduced by adding blocks of the same mass to both arms symmetrically (the blocks carried by the arms of the tuning fork must be symmetrical). The vibration period T for this loaded tuning fork is given by the following formula similar to (3)

$$T^2 = B(m + m_0) \ , \tag{6}$$

where B is a constant, which depends on the mechanical properties, size and shape of the tuning fork. m_0 is a constant related to the effective mass of each vibrating arm. Using formula (6), various tuning fork sensors can be made, such as liquid density sensors and liquid level sensors. By measuring the resonance frequency of the tuning fork, the density or level of the liquid in the tuning fork tube can be obtained.