4. Experimental Objectives

1) Determine the amplitude and the phase-frequency characteristics of forced vibration
2) Study the influence of damping coefficient on forced vibration and observe resonant vibration phenomenon
3) Determine the properties of a moving object using frequency-flash method.

5. Experimental Procedures

1) Measure the natural frequency $\omega_0$ of the system

The natural frequency is the frequency at which the system vibrates freely without damping.

Firstly, confirm that the pointer of the angle dial is at the position of $0^\circ$, and turn the damping switch to the lowest. At this time, the balance wheel does not pass through the magnetic force line.

Then, turn on the power of the electrical unit. Manually turn the balance wheel to “110°-140°” and release. Read the value of the balance wheel swinging 10 cycles (periods) from the electrical unit. Calculate the natural frequency $\omega_0$.

2) Determine the damping coefficient $\beta$

The damping vibration is carried out under the condition that the driving force is zero. When doing this experiment, the power of the motor must be turned off, and the angle dial pointer should be placed at $0^\circ$.

Since there is no driving force, the solution of motion equation (1) is $\theta = \theta_0 e^{-\beta t} \cos(\omega t + \alpha)$, correspondingly, $\theta_1 = \theta_0 e^{-\beta t}$, $\theta_2 = \theta_0 e^{-\beta(2t)}$, ..., $\theta_n = \theta_0 e^{-\beta(nt)}$.

Using $\ln \frac{\theta_i}{\theta_j} = \ln \frac{\theta_0 e^{-\beta(i)}}{\theta_0 e^{-\beta(j)}} = (i - j) \beta T$ (where $\theta_i$ and $\theta_j$ are the amplitudes of the i-th and j-th vibrations, respectively, and $T$ is the average value of the damped vibration period), we can get $\beta$ value.

Adjust the magnet position up and down, and then fix it at a certain position. It cannot be changed arbitrarily during the experiment. Note: The damping should not be too small, otherwise the amplitude of the forced vibration will be too large or even exceed $180^\circ$, which will damage the apparatus!

Manually turn the balance wheel to between $120^\circ$ - $140^\circ$ ($\theta_0$), and then release it. The balance wheel will take damping vibration now. Record the amplitudes $\theta_0$, $\theta_1$, $\theta_2$, ..., $\theta_n$, of the thereafter every vibration cycles in sequence, and at the same time recording the corresponding period value (the total time for all cycles, then take an average). If some data of amplitudes are missed to record, just ignore it.

Repeat the above procedure multiple times. Do average operation for the amplitudes of the same cycle order. Finally average the period data of all experiment times.

As shown in the table below, the averaged amplitude data can be processed by the difference
method to obtain the $\beta$ value.

Table 1 Data table for $\beta$ value calculation (The position of the damping switch is _____ )

<table>
<thead>
<tr>
<th>Amplitude ($^\circ$)</th>
<th>Amplitude ($^\circ$)</th>
<th>$\ln \frac{\theta_i}{\theta_{i+5}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>$\theta_1$</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$\theta_2$</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>$\theta_3$</td>
<td></td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>$\theta_4$</td>
<td></td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>$\theta_5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\theta_6$</td>
<td>Average:</td>
</tr>
</tbody>
</table>

Average value of period: $\bar{T} = ____ s$ (the average for all periods of all cycles)

Using $5\beta T = \ln \frac{\theta_i}{\theta_{i+5}}$, we can calculate $\beta$ value.

3) Measure amplitude-frequency and phase-frequency characteristics of forced vibration

Turn on the motor and flash switch. When the forced vibration is stable, the period of the balance wheel vibration should be the same as the period of the motor.

Change the motor speed, i.e. the frequency of the driving torque. Read the amplitude value of the balance wheel (At this time, the first term of the equation solution tends to zero at this time, only the second term exists). Record the period value of the vibration. Use the flash to measure the phase difference $\varphi$ between the displacement of the forced vibration and driving force.

The change range of the motor speed can be determined by controlling $\Delta \varphi$ at around 10°. Since the curve changes greatly near the resonance point, the measurement data needs to be relatively dense. At this time, a small change in the motor speed will cause a large change in $\Delta \varphi$. The relative error of measurement increases when the phase difference $\varphi$ is around 0° and 180°. So the experimental data of phase difference $\varphi$ is selected between 30° and 150°.

Taking $\omega/\omega_0$ as the abscissa and amplitude $\theta$ as the ordinate, make the amplitude-frequency curve;

Taking $\omega/\omega_0$ as the abscissa and phase difference $\varphi$ as the ordinate, make the phase-frequency curve.

These two curves fully reflect the characteristics of the vibration system.

Table 2 Data of amplitude frequency characteristic and phase frequency characteristic

<table>
<thead>
<tr>
<th>$10T/s$</th>
<th>$\omega = \frac{2\pi}{T}$</th>
<th>$\varphi/(^\circ)$</th>
<th>$\theta/(^\circ)$</th>
<th>$\omega / \omega_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4) Repeat the steps of 2) and 3) by changing different damping

6. An example of data recording and processing

Note: Following data are for reference only, not the criteria for apparatus performance:

1) Measure the natural frequency \( \omega_0 \) of the system

Turn the damping switch to 2.0 cm position. At this time, the balance wheel does not pass through the magnetic force lines. Turn the balance wheel to 130° and release. Measure the average time of 10 cycles of the balance wheel. Calculate the natural frequency \( \omega_0 \) of the system.

\[
T = 1.119 \text{ s}, \quad \omega_0 = \frac{2\pi}{1.119} = 5.615 \text{ s}^{-1}
\]

2) Determine the damping coefficient \( \beta \)

Turn off the power of the motor, and place the angle dial pointer at 0°.

Set \( \theta_0 \) at 130° by hand turning the balance wheel to 130°. Read the amplitude values of following cycles of the damping vibration, \( \theta_0, \theta_1, \theta_2, \ldots, \theta_n \).

Table 1 Data table for \( \beta \) value calculation (The position of the damping switch is 3.8 cm)

<table>
<thead>
<tr>
<th>Amplitude (°)</th>
<th>Amplitude (°)</th>
<th>( \ln \frac{\theta_i}{\theta_{i+4}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_0 )</td>
<td>130</td>
<td>( \theta_4 ) 64</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>109</td>
<td>( \theta_5 ) 52</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>92</td>
<td>( \theta_6 ) 44</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>76</td>
<td>( \theta_7 ) 36</td>
</tr>
</tbody>
</table>

Average: 0.734

\[
\bar{T} = 1.118 \text{ s}
\]

Using \( 4\beta T = \ln \frac{\theta_i}{\theta_{i+4}} \), we got \( \beta_i = \frac{0.734}{4 \times 1.118} = 0.164 \text{s}^{-1} \).

3) Measure amplitude-frequency and phase-frequency characteristics of forced vibration

Change the motor speed, i.e. driving torque frequency. When the forced vibration is stable, read the amplitude value of the balance wheel, and use the flash to measure the phase difference \( \phi \) between the forced vibration displacement and the driving force (the change of the motor speed is controlled to achieve \( \Delta \phi \) around 10°).

The data should be relatively dense near the resonance point due to the large curve change. At this time, a small change in the motor speed will cause a large change in \( \Delta \phi \).

Table 2 Data of amplitude frequency and phase frequency characteristics The position of damping switch is 3.8 cm
<table>
<thead>
<tr>
<th>$T/s$</th>
<th>$\omega = \frac{2\pi}{T}/s^{-1}$</th>
<th>$\varphi/(^\circ)$</th>
<th>$\theta/(^\circ)$</th>
<th>$\frac{\omega}{\omega_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.050</td>
<td>5.984</td>
<td>157</td>
<td>55</td>
<td>1.066</td>
</tr>
<tr>
<td>1.060</td>
<td>5.928</td>
<td>155</td>
<td>63</td>
<td>1.056</td>
</tr>
<tr>
<td>1.070</td>
<td>5.872</td>
<td>150</td>
<td>73</td>
<td>1.046</td>
</tr>
<tr>
<td>1.080</td>
<td>5.818</td>
<td>144</td>
<td>86</td>
<td>1.036</td>
</tr>
<tr>
<td>1.090</td>
<td>5.764</td>
<td>136</td>
<td>101</td>
<td>1.027</td>
</tr>
<tr>
<td>1.100</td>
<td>5.712</td>
<td>126</td>
<td>120</td>
<td>1.017</td>
</tr>
<tr>
<td>1.110</td>
<td>5.660</td>
<td>112</td>
<td>139</td>
<td>1.008</td>
</tr>
<tr>
<td>1.115</td>
<td>5.635</td>
<td>102</td>
<td>146</td>
<td>1.004</td>
</tr>
<tr>
<td>1.120</td>
<td>5.610</td>
<td>94</td>
<td>150</td>
<td>0.999</td>
</tr>
<tr>
<td>1.125</td>
<td>5.585</td>
<td>84</td>
<td>149</td>
<td>0.995</td>
</tr>
<tr>
<td>1.130</td>
<td>5.560</td>
<td>76</td>
<td>145</td>
<td>0.990</td>
</tr>
<tr>
<td>1.140</td>
<td>5.512</td>
<td>60</td>
<td>128</td>
<td>0.982</td>
</tr>
<tr>
<td>1.150</td>
<td>5.464</td>
<td>49</td>
<td>110</td>
<td>0.973</td>
</tr>
<tr>
<td>1.160</td>
<td>5.416</td>
<td>41</td>
<td>94</td>
<td>0.965</td>
</tr>
<tr>
<td>1.170</td>
<td>5.370</td>
<td>35</td>
<td>81</td>
<td>0.956</td>
</tr>
<tr>
<td>1.180</td>
<td>5.325</td>
<td>30</td>
<td>71</td>
<td>0.948</td>
</tr>
<tr>
<td>1.190</td>
<td>5.280</td>
<td>26</td>
<td>63</td>
<td>0.940</td>
</tr>
<tr>
<td>1.200</td>
<td>5.236</td>
<td>24</td>
<td>57</td>
<td>0.932</td>
</tr>
</tbody>
</table>

4) Repeat the steps of 2) and 3) by changing different damping

Table 3 Data table for $\beta$ value calculation (Position of damping switch 3.9 cm)

<table>
<thead>
<tr>
<th>Amplitude (°)</th>
<th>Amplitude (°)</th>
<th>$\ln\frac{\theta_i}{\theta_{i+4}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>130</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>106</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>86</td>
<td>$\theta_6$</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>69</td>
<td>$\theta_7$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\bar{T} = 1.119$ s
Using $4\beta T = \ln \frac{\theta_i}{\theta_{i+4}}$, we got $\beta_2 = \frac{0.865}{4 \times 1.119} = 0.193 \text{s}^{-1}$.

Table 4 Data table for $\beta$ value calculation (Position of damping switch 4.1 cm)

<table>
<thead>
<tr>
<th>Amplitude (°)</th>
<th>Amplitude (°)</th>
<th>$\ln \frac{\theta_i}{\theta_{i+3}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>130</td>
<td>54</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>97</td>
<td>39</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>73</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average 0.904</td>
</tr>
</tbody>
</table>

$\bar{T} = 1.118 \text{ s}$

Using $3\beta T = \ln \frac{\theta_i}{\theta_{i+3}}$, we got $\beta_3 = \frac{0.904}{3 \times 1.118} = 0.270 \text{s}^{-1}$.

Table 5 Data table for $\beta$ value calculation (Position of damping switch 4.5 cm)

<table>
<thead>
<tr>
<th>Amplitude (°)</th>
<th>Amplitude (°)</th>
<th>$\ln \frac{\theta_i}{\theta_{i+3}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$</td>
<td>130</td>
<td>36</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>84</td>
<td>23</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>56</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average 1.322</td>
</tr>
</tbody>
</table>

$\bar{T} = 1.120 \text{ s}$

Using $3\beta T = \ln \frac{\theta_i}{\theta_{i+3}}$, we got $\beta_4 = \frac{1.322}{3 \times 1.120} = 0.393 \text{s}^{-1}$.

(Note: the measurement data of amplitude-frequency characteristic and phase-frequency characteristic under different damping are omitted here).

5) Amplitude-frequency and phase-frequency characteristic under different damping

Taking $\omega/\omega_0$ as the abscissa and amplitude $\theta$ as the ordinate, make the amplitude-frequency characteristic curve;

Taking $\omega/\omega_0$ as the abscissa and the phase difference $\varphi$ as the ordinate, make the phase frequency characteristic curve.
It can be seen from the above figure that when the driving force frequency is equal to the natural frequency, the amplitude reaches the maximum, i.e., the resonance state is reached, and the phase difference at this time is $\pi/2$. 