4. Experimental Objectives

1) Determine the amplitude and the phase-frequency characteristics of forced vibration
2) Study the influence of damping coefficient on forced vibration and observe resonant vibration phenomenon
3) Determine the properties of a moving object using frequency-flash method.

5. Experimental Procedures

1) Measure the natural frequency ω_0 of the system

The natural frequency is the frequency at which the system vibrates freely without damping.

Firstly, confirm that the pointer of the angle dial is at the position of 0°, and turn the damping switch to the lowest. At this time, the balance wheel does not pass through the magnetic force line.

Then, turn on the power of the electrical unit. Manually turn the balance wheel to "110°-140°" and release. Read the value of the balance wheel swinging 10 cycles (periods) from the electrical unit. Calculate the natural frequency ω_0.

2) Determine the damping coefficient β

The damping vibration is carried out under the condition that the driving force is zero. When doing this experiment, the power of the motor must be turned off, and the angle dial pointer should be placed at 0°.

Since there is no driving force, the solution of motion equation (1) is $\theta = \theta_0 e^{-\beta t}\cos(\omega_f t + \alpha)$,

correspondingly, $\theta_1 = \theta_0 e^{-\beta t}$, $\theta_2 = \theta_0 e^{-\beta(2t)}$, …, $\theta_n = \theta_0 e^{-\beta(nt)}$.

Using $\ln\frac{\theta_i}{\theta_j} = \ln\frac{\theta_0 e^{-\beta(it)}}{\theta_0 e^{-\beta(jt)}} = (i-j)\beta T$ (where θ_i and θ_j are the amplitudes of the i-th and j-th vibrations, respectively, and T is the average value of the damped vibration period), we can get β value.

Adjust the magnet position up and down, and then fix it at a certain position. It cannot be changed arbitrarily during the experiment. **Note:** The damping should not be too small, otherwise the amplitude of the forced vibration will be too large or even exceed 180°, which will damage the apparatus!

Manually turn the balance wheel to between 120° - 140° (θ_0), and then release it. The balance wheel will take damping vibration now. Record the amplitudes θ_0, θ_1, θ_2, …, θ_n, of the thereafter every vibration cycles in sequence, and at the same time recording the corresponding period value (the total time for all cycles, then take an average). If some data of amplitudes are missed to record, just ignore it.

Repeat the above procedure multiple times. Do average operation for the amplitudes of the same cycle order. Finally average the period data of all experiment times.

As shown in the table below, the averaged amplitude data can be processed by the difference method to obtain the β value.

Table 1 Data table for β value calculation (The position of the damping switch is _____)

Amplitude (°)		Amplitude (°)		$\ln\frac{\theta_i}{\theta_{i+5}}$
θ_0		θ_5		
θ_1		θ_6		
θ_2		θ_7		
θ_3		θ_8		
θ_4		θ_9		
				Average:

Average value of period: $\overline{T}$ =___s (the average for all periods of all cycles)

Using $5\beta T = \ln\frac{\theta_i}{\theta_{i+5}}$, we can calculate β value.

3) Measure amplitude-frequency and phase-frequency characteristics of forced vibration

Turn on the motor and flash switch. When the forced vibration is stable, the period of the balance wheel vibration should be the same as the period of the motor.

Change the motor speed, i.e. the frequency of the driving torque. Read the amplitude value of the balance wheel (At this time, the first term of the equation solution tends to zero at this time, only the second term exists). Record the period value of the vibration. Use the flash to measure the phase difference φ between the displacement of the forced vibration and driving force.

The change range of the motor speed can be determined by controlling Δφ at around 10°. Since the curve changes greatly near the resonance point, the measurement data needs to be relatively dense. At this time, a small change in the motor speed will cause a large change in Δφ. The relative error of measurement increases when the phase difference φ is around 0 ° and 180°. So the experimental data of phase difference φ is selected between 30° and 150°.

Taking ω/ω_0 as the abscissa and amplitude θ as the ordinate, make the amplitude-frequency curve;

Taking ω/ω_0 as the abscissa and phase difference φ as the ordinate, make the phase-frequency curve.

These two curves fully reflect the characteristics of the vibration system.

Table 2 Data of amplitude frequency characteristic and phase frequency characteristic

$10T\,/\,\mathrm{s}$	$\omega = \frac{2\pi}{T}\,/\,\mathrm{s}^{-1}$	$\varphi\,/(°)$	$\theta\,/(°)$	ω/ω_0

4) Repeat the steps of 2) and 3) by changing different damping

6. An example of data recording and processing

Note: Following data are for reference only, not the criteria for apparatus performance:

1) Measure the natural frequency ω_0 of the system

Turn the damping switch to 2.0 cm position. At this time, the balance wheel does not pass through the magnetic force lines. Turn the balance wheel to 130° and release. Measure the average time of 10 cycles of the balance wheel. Calculate the natural frequency ω_0 of the system.

$T = 1.119$ s, $\omega_0 = 2\pi / 1.119 = 5.615\ s^{-1}$

2) Determine the damping coefficient β

Turn off the power of the motor, and place the angle dial pointer at 0°.

Set θ_0 at 130° by hand turning the balance wheel to 130°. Read the amplitude values of following cycles of the damping vibration, θ_0, θ_1, θ_2, …, θ_n.

Table 1 Data table for β value calculation (The position of the damping switch is 3.8 cm)

Amplitude (°)		Amplitude (°)		$\ln\frac{\theta_i}{\theta_{i+4}}$
θ_0	130	θ_4	64	0.709
θ_1	109	θ_5	52	0.740
θ_2	92	θ_6	44	0.738
θ_3	76	θ_7	36	0.747
				Average: 0.734

$\overline{T} =$ 1.118 s

Using $4\beta T = \ln\frac{\theta_i}{\theta_{i+4}}$, we got $\beta_1 = \frac{0.734}{4\times 1.118} = 0.164\text{s}^{-1}$.

3) Measure amplitude-frequency and phase-frequency characteristics of forced vibration

Change the motor speed, i.e. driving torque frequency. When the forced vibration is stable, read the amplitude value of the balance wheel, and use the flash to measure the phase difference φ between the forced vibration displacement and the driving force (the change of the motor speed is controlled to achieve Δφ around 10°).

The data should be relatively dense near the resonance point due to the large curve change. At this time, a small change in the motor speed will cause a large change in Δφ.

Table 2 Data of amplitude frequency and phase frequency characteristics The position of damping switch is 3.8 cm

T / s	$\omega = \frac{2\pi}{T}$ / s^{-1}	φ /(°)	θ /(°)	ω / ω_0
1.050	5.984	157	55	1.066
1.060	5.928	155	63	1.056
1.070	5.872	150	73	1.046
1.080	5.818	144	86	1.036
1.090	5.764	136	101	1.027
1.100	5.712	126	120	1.017
1.110	5.660	112	139	1.008
1.115	5.635	102	146	1.004
1.120	5.610	94	150	0.999
1.125	5.585	84	149	0.995
1.130	5.560	76	145	0.990
1.140	5.512	60	128	0.982
1.150	5.464	49	110	0.973
1.160	5.416	41	94	0.965
1.170	5.370	35	81	0.956
1.180	5.325	30	71	0.948
1.190	5.280	26	63	0.940
1.200	5.236	24	57	0.932

4) Repeat the steps of 2) and 3) by changing different damping

Table 3 Data table for β value calculation (Position of damping switch 3.9 cm)

Amplitude (°)		Amplitude(°)		$\ln\frac{\theta_i}{\theta_{i+4}}$
θ_0	130	θ_4	56	0.842
θ_1	106	θ_5	44	0.880
θ_2	86	θ_6	36	0.871
θ_3	69	θ_7	29	0.867
				Average 0.865

$\overline{T}$ = 1.119 s

Using $4\beta T = \ln\frac{\theta_i}{\theta_{i+4}}$, we got $\beta_2 = \frac{0.865}{4\times1.119} = 0.193\text{s}^{-1}$.

Table 4 Data table for β value calculation (Position of damping switch 4.1 cm)

Amplitude (°)		Amplitude (°)		$\ln\frac{\theta_i}{\theta_{i+3}}$
θ_0	130	θ_3	54	0.878
θ_1	97	θ_4	39	0.911
θ_2	73	θ_5	29	0.923
				Average 0.904

$\overline{T}$ = 1.118 s

Using $3\beta T = \ln\frac{\theta_i}{\theta_{i+3}}$, we got $\beta_3 = \frac{0.904}{3\times1.118} = 0.270\text{s}^{-1}$.

Table 5 Data table for β value calculation (Position of damping switch 4.5 cm)

Amplitude (°)		Amplitude (°)		$\ln\frac{\theta_i}{\theta_{i+3}}$
θ_0	130	θ_3	36	1.284
θ_1	84	θ_4	23	1.295
θ_2	56	θ_5	14	1.386
				Average 1.322

$\overline{T}$ = 1.120 s

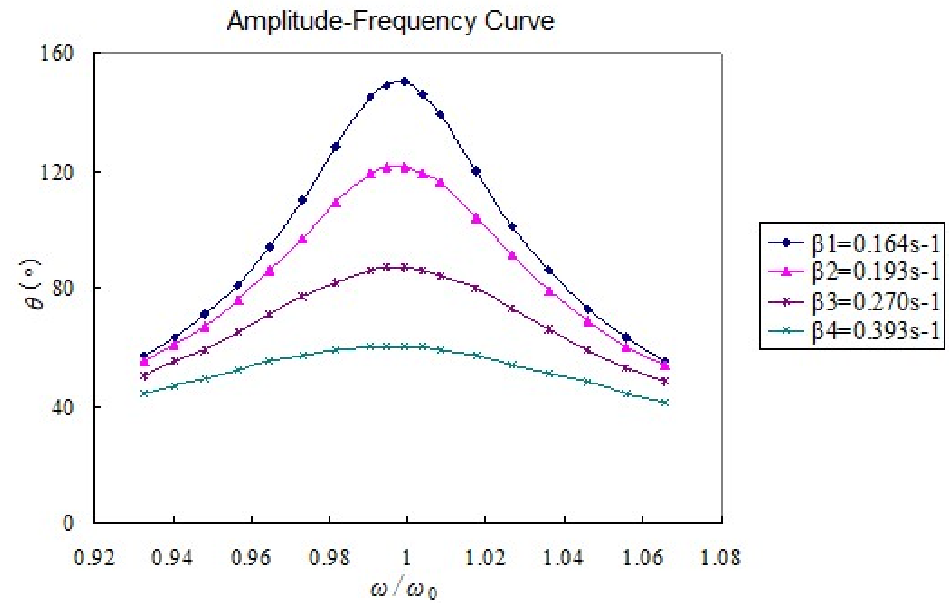
Using $3\beta T = \ln\frac{\theta_i}{\theta_{i+3}}$, we got $\beta_4 = \frac{1.322}{3\times1.120} = 0.393\text{s}^{-1}$.

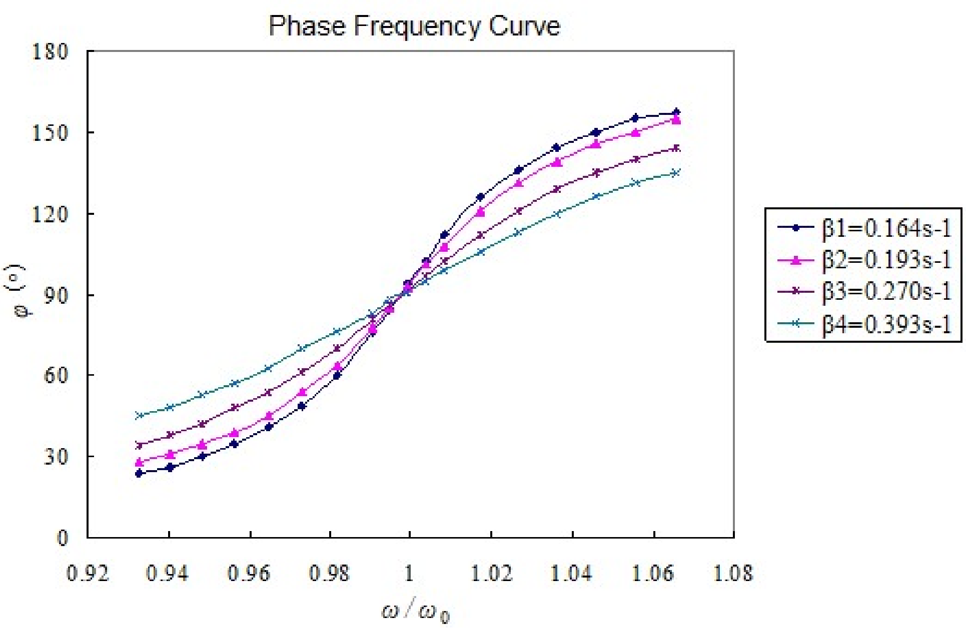
(Note: the measurement data of amplitude-frequency characteristic and phase-frequency characteristic under different damping are omitted here).

5) Amplitude-frequency and phase-frequency characteristic under different damping

Taking ω/ω_0 as the abscissa and amplitude θ as the ordinate, make the amplitude-frequency characteristic curve;

Taking ω/ω_0 as the abscissa and the phase difference φ as the ordinate, make the phase frequency characteristic curve





It can be seen from the above figure that when the driving force frequency is equal to the natural frequency, the amplitude reaches the maximum, i.e., the resonance state is reached, and the phase difference at this time is $\pi/2$.