## 2. Theory

A three-string pendulum consists of a circular disk of uniform mass distribution, which is hung under a small circular disk using three small strings symmetrically. The three hanging points of each disk form an equilateral triangle. As shown in Figure 1, when the disks are leveled and the three strings are in equal length, the lower disk can swing around the axis ($O_1O_2$) perpendicular to the surfaces of the disks. The period of swing is related to the moment of inertia of the lower disk (including any objects placed on the disk). The three-string pendulum method is to determine the moment of inertia of an object with known mass by measuring the swinging period.

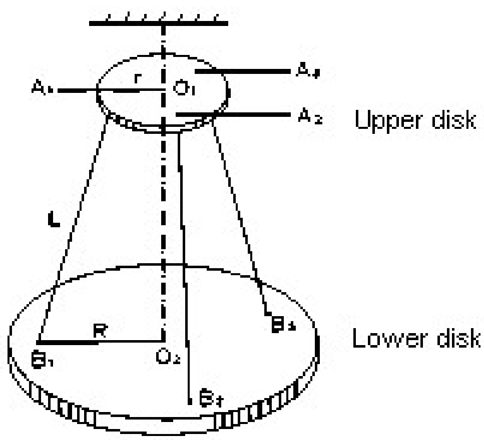


Figure 1 Schematic of a three-string torsional pendulum

From the deviation shown in Appendix 1, when the angle of swing is small, the three hanging strings are equal length with equal tension, the two disks are parallel, and the lower disk rotates only around axis $O_1O_2$, the moment of inertia $J_0$ of the lower disk $B$ around axis $O_1O_2$ is:

$$J_0 = \frac{m_0 gRr}{4\pi^2 H} T_0^2 \tag{1}$$

where $m_0$ is the mass of the lower disk, $r$ is the distance between hanging point $A_i$ and disk center $O_1$, $R$ is the distance between hanging point $B_i$ and disk center $O_2$ (note: $r$ and $R$ are not the radii of the disks), $H$ is the vertical spacing between the two disks, and $T_0$ is the period of swing of disk $B$.

To measure the moment of inertia $J_1$ of an object with mass $m$, the object needs to be placed on the lower disk and the period of swing $T$ of the pendulum should be measured. Thus, the total moment of inertia around axis $O_1O_2$ is:

$$J_1 = J + J_0 = \frac{(m + m_0)gRr}{4\pi^2 H} T^2 \tag{2}$$

Therefore, the moment of inertia of the object under test is:

$$J = \frac{(m + m_0)gRr}{4\pi^2 H} T^2 - J_0 \tag{3}$$

**Note:** the object under test on disk $B$ needs to be orientated so that its center of mass coincides with the center of the disk.

This apparatus can also be used to verify the parallel axis theorem. If a small cylinder of known mass $m_2$ and diameter $D$ is placed on the center of disk $B$, the moment of inertia of the cylinder can be calculated as

$$J_2 = \frac{1}{8} m_2 D^2 \tag{4}$$

Next, the center of mass of the cylinder is pulled away from the center of the disk by a distance $d$. To balance the disk, an identical cylinder should be placed symmetrically on the disk, as shown in Figure 2. Now, the moment of inertia of the two cylinders around axis $O_1O_2$ becomes $J'_2$. According to the parallel axis theorem, we get:

$$m_2 d^2 = \frac{J_2'}{2} - J_2 \tag{5}$$

Therefore, $d$ can be calculated from equation (5) in comparison with the measured value.

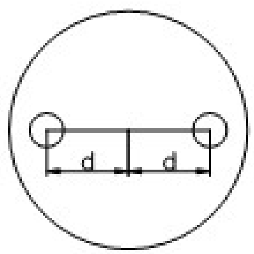


Figure 2 Cylinders' placement