1. Experimental Contents

1) Measure moment of inertia using three-string pendulum method
2) Measure the moment of inertia of two objects of same mass with different mass distribution
3) Verify the parallel axis theorem of moment of inertia

2. Experimental Procedures

A. Adjustment of three-string pendulum

1) Level the upper disk: place the level on the suspension arm while adjusting the base feet to level the upper disk.
2) Level the lower disk: place the level at the center of the lower disk while releasing the tightening bolt of string adjustment and adjusting the string adjustment knob to level the lower disk.

B. Adjustment of laser and timer

a) Place the photo-receiver at an appropriate location while adjusting the laser position to level the laser at the same height as the receiver. Turn on the power, and adjust the laser beam to let it enter the small hole of the receiver. At this moment, the indicator of the timer should be dim. ***Warning:*** *do not stare into the laser source*.

b) Adjust the upper disk, and let one string close to the laser beam while gently rotating the upper disk by 5° so that the string can pass through the beam.

c) Preset the counting number (20 or 40e as the half-cycles, since the string will block the laser beam twice for each cycle).

C. Measurement of moment of inertia of lower disk

a) Calculate r and R of the upper and lower disks per Figure 4, $r = \frac{\sqrt{3}}{3}a$. Use a caliper to measure diameter D_1 of the lower disk.

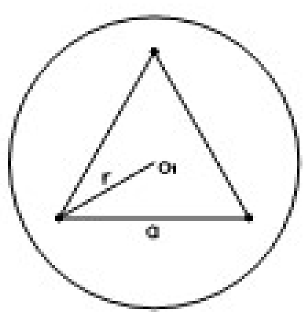

Figure 4 Schematic drawing of disk

b) Measure the spacing, H, between the upper and lower disks with a ruler.

c) Measure the mass, m_0, of the lower disk.

d) Measure the period of swing of the lower disk. To eliminate other motions except the swinging motion of the lower disk, the upper disk is set to rotate freely around the axis

O_1O_2. To begin with, let the lower disk stationary and then gently rotate the upper disk. Through the tension of the three hanging strings, the lower disk can undertake pure swing motion (swing angle < 5°). Record the elapsed time of 10 or 20 cycles.

e) Calculate the moment of inertia of the lower disk, J_0.

f) Measurement of moment of inertia of lower disk

D. Measurement of moment of inertia of lower disk plus ring sample

a) Place the ring sample on the lower disk and align its center with the disk center.

b) Measure the period of swing of the disk plus the ring, T_1.

c) Measure the mass of the ring m_1, and the inside and outside diameters D_{in} and D_{out} of the ring sample.

d) Calculate the moment of inertia of the disk plus ring, J_1, and the moment of inertia of the ring sample, J_{m1}.

E. Measurement of moment of inertia of lower disk plus disc sample

a) Place the disc sample on the lower disk and align its center with the lower disk center.

b) Measure the period of swing of the lower disk plus the disc, T_3.

c) Measure the mass of the disc sample m_3 and its diameter D_{disc}.

d) Calculate the moment of inertia of the lower disk plus disc sample, J_3, and the moment of inertia of the disc sample, J_{m3}.

F. Compare the moment of inertia of both samples. Compare the theoretical value of moment of inertia of the lower disk, the ring, and the disc with the corresponding measurement value, respectively, and calculate the percentage error of experiment.

G. Verification of parallel axis theorem

a) Place two identical cylinders on the lower disk in symmetry as shown in Figure 2.

b) Measure period of swing, T_2.

c) Measure the diameter D_{cyl} of the cylinder and the total mass $2m_2$ of the two cylinders.

d) Calculate the moment of inertia, J'_2, of the two cylinders with an equal distance of d of their mass centers from the axis O_1O_2.

e) Use formula $J=mD^2/8$ to calculate the moment of inertia of a single cylinder on the axis O_1O_2, J_2.

f) Use formula (5) $md^2=J'_2/2-J_2$ to calculate d and compare with measured value.

3. Example of Data Recording and Processing

Note: following data are for reference purposes only, not the criteria for apparatus performance.

Table 1 Measured period of swing

		Lower Disk m_0=614.93 g	Ring Sample m_1=231.32 g	Cylindrical Samples $2m_2$=239.95 g	Disc Sample m_3=234.38 g
Period (n)		10	10	10	10
Time (s)	1	17.411	16.472	16.378	16.207
	2	17.370	16.520	16.400	16.240
	3	17.386	16.569	16.431	16.203
	4	17.388	16.505	16.406	16.183
Average (s)		17.389	16.516	16.404	16.208
Average period (s) $T_i = \bar{t}/n$		T_0=1.7389	T_1=1.6516	T_2=1.6404	T_3=1.6208

Table 2 Parameters of upper and lower disks

		D_1 (cm)	H (cm)	a (cm)	b (cm)	$R = \frac{\sqrt{3}}{3}\bar{a}$ (cm)	$r = \frac{\sqrt{3}}{3}\bar{b}$ (cm)
	1	16.794	51.90	13.90	5.31	8.025	3.060
	2	16.792	51.89	13.91	5.30		
	3	16.790	51.91	13.89	5.29		
Mean (cm)		16.792	51.90	13.90	5.30		

Table 3 Geometrical parameters of ring, disc, and cylindrical samples

		D_{in} (cm)	D_{out} (cm)	D_{disc} (cm)	D_{cyl} (cm)	$2d$ (cm)
	1	6.014	11.990	11.990	2.999	9.00
	2	6.012	11.992	11.992	3.000	
	3	6.016	11.994	11.994	3.001	
Mean (cm)		6.014	11.992	11.992	3.000	

Data processing:

1. Calculate the moment of inertia of the lower disk, the ring and the disc. Compare the moment of inertia between a ring and a disc of same mass. Explain the relationship between moment of inertia and mass distribution.

1) Measured moment of inertia:

The lower disk:

$$J_0 = \frac{gRr}{4\pi^2 H} m_0 T_0^2 = \frac{979.4\times 8.025\times 3.060}{4\pi^2\times 51.90}\times 614.93\times 1.7389^2$$

$$= 2.183\times 10^4\ g\cdot cm^2 = 2.183\times 10^{-3} kg\cdot m^2$$

The lower disk plus ring:

$$J_1 = \frac{gRr}{4\pi^2 H}(m_0 + m_1)T_1^2 = \frac{979.4\times 8.025\times 3.060}{4\pi^2\times 51.90}\times (614.93+231.32)\times 1.6516^2$$

$$= 2.710\times 10^4 g\cdot cm^2 = 2.710\times 10^{-3} kg\cdot m^2$$

The lower disk plus disc:

$$J_3 = \frac{gRr}{4\pi^2 H}(m_0 + m_3)T_3^2 = \frac{979.4\times 8.025\times 3.060}{4\pi^2\times 51.90}\times (614.93+234.38)\times 1.6208^2$$

$$= 2.619\times 10^4 g\cdot cm^2 = 2.619\times 10^{-3} kg\cdot m^2$$

The ring:

$$J_{m1} = J_1 - J_0 = 2.710\times 10^{-3} - 2.183\times 10^{-3} = 0.527\times 10^{-3}\ kg\cdot m^2$$

The disc:

$$J_{m3} = J_3 - J_0 = 2.619\times 10^{-3} - 2.183\times 10^{-3} = 0.436\times 10^{-3} kg\cdot m^2$$

2) Theoretical values:

The lower disk:

$$J_0^{/} = \frac{1}{8} m_0 D_1^2 = \frac{1}{8}\times 614.93\times 16.792^2 = 2.167\times 10^4 g\cdot cm^2 = 2.167\times 10^{-3} kg\cdot m^2$$

The ring:

$$J_{m1}^{/} = \frac{1}{8} m_1 \left(D_{in}^2 + D_{out}^2\right) = \frac{1}{8}\times 231.32\times \left(6.014^2 + 11.992^2\right)\times 10^{-7} = 0.5204\times 10^{-3} kg\cdot m^2$$

The disc:

$$J_{m3}^{/} = \frac{1}{8}\times m_3\times D^2_{disk} = \frac{1}{8}\times 234.38\times 11.992^2\times 10^{-7} = 0.4213\times 10^{-3} kg\cdot m^2$$

Conclusion: even with similar mass, the moment of inertia of a ring is larger than that of a disc, meaning that the farther mass distribution from the axis, the larger moment of inertia.

3) Errors

The lower disk: $E_0 = \frac{|J_0^{'} - J_0|}{J_0^{'}} = \frac{2.183 - 2.167}{2.167} = 0.74\%$

The ring: $E_{m1} = \frac{|J_{m1}^{'} - J_{m1}|}{J_{m1}^{'}} = \frac{0.527 - 0.5204}{0.5204} = 1.3\%$

The disc: $E_{m3} = \frac{|J_{m3}^{'} - J_{m3}|}{J_{m3}^{'}} = \frac{0.436 - 0.4213}{0.4213} = 3.5\%$

Conclusion: experimental and theoretical values are within an error range of ± 5%.

2. Verification of parallel axis theorem

If two cylindrical samples are placed on the lower disk symmetrically from axis O_1O_2 with an equal distance of d from the center of the disk, the total moment of inertia of the two cylinders and the lower disk is:

$$J_2^{'} + J_0 = \frac{gRr}{4\pi^2 H}(m_0 + 2m_2)T_2^2 = \frac{979.4 \times 8.025 \times 3.060}{4\pi^2 \times 51.90} \times (614.93 + 239.95) \times 1.6404^2$$
$$= 2.700 \times 10^4\, g \cdot cm^2 = 2.700 \times 10^{-3}\, kg \cdot m^2$$

The moment of inertia of the two cylinders around axis O_1O_2 is:

$$J_2^{'} = 2.700 \times 10^{-3} - 2.183 \times 10^{-3} = 0.517 \times 10^{-3}\, kg \cdot m^2$$

The moment of inertia of a single cylinder at the axis is:

$$J_2 = \frac{1}{8} \times m_2 D_{cyl}^2 = \frac{1}{8} \times \frac{239.95}{2} \times 3.000^2 \times 10^{-7} = 0.01350 \times 10^{-3}\, kg \cdot m^2 .$$

Base on formula (5) $md^2 = \frac{J_2^{'}}{2} - J_2$, distance d is calculated as:

$$d = \sqrt{\frac{1}{m_2}\left(\frac{J_2^{'}}{2} - J_2\right)} = \sqrt{\frac{1}{119.975 \times 10^{-3}} \times \left(\frac{0.517}{2} - 0.01350\right) \times 10^{-3}} = 0.04519\, m$$

The measured value: $d^{'} = \frac{9.00}{2} \times 10^{-2} = 0.045\ m$

Percentage error: $E_d = \frac{|d - d^{'}|}{d^{'}} = \frac{0.04519 - 0.045}{0.045} = 0.4\%$

Conclusion: the parallel axis theorem is verified within an acceptable error range.