1. Experimental Contents

1) Learn how to measure length and time
2) Learn how to measure shear modulus using torsional pendulum
3) Learn how to measure rotational moment of inertia of various objects
4) Verify perpendicular/parallel-axis theorems

2. Experimental Procedure

A. Measure shear modulus of steel wire

1) Measure the mass of the ring using a balance, measure height d, outer diameter c, and inner diameter b of the ring using a caliper, and measure diameter $2R$ of the wire using a micrometer or a caliper.
2) Fix one end of the wire to the claw at clamping point O' while fixing the other end of the wire to the top of the support at pivot point O. Hang the claw and then measure the distance, L, between O and O' using a ruler.
3) Since the wire is long enough, the condition of α<<1° is met. The angular amplitude Φ_i of free swing can take a large value of 2π. By placing the rigid ring horizontally onto the claw while manually rotating the clamp by a certain angle and releasing it back to its original position quickly, the claw with the horizontal ring takes periodical swings. Use the Hall switch and a stopwatch to measure the swing period T_1, respectively.
4) Change the ring from horizontal state to vertical state, and repeat Step 3) to measure the swing period T_2.
5) Calculate the shear modulus G of the steel wire, and compare the result using the digital counter/timer to that using a stopwatch.

B. Measure shear modulus of copper wire, and verify parallel and perpendicular axis theorems

1) Measure the shear modulus of a copper wire, and compare it with that of the steel wire.
2) Measure the rotational moment of inertia of a square rigid bar and a round rigid rod rotating around the steel wire as a torsional pendulum, and compare with the theoretical values.
3) Place two small steel balls onto both ends of the claw to verify the parallel axis theorem. Using the measured torsional coefficient of the steel wire to verify the perpendicular axis theorem by measuring the rotational moments of inertia of the rigid ring positioned horizontally and vertically on the claw rotating around the steel wire.

3. Example of Data Recording and Processing

Note: following data are for reference purposes only, not the criteria for apparatus performance.

1. Parameter measurement of samples

Table 1 Measured parameters of ring, square bar, round bar and small ball (unit: cm)

No.	c	b	d_{square}	d_{round}	$2R_{ball}$
1	10.992	7.974	11.998	11.990	2.000

2	10.990	7.990	12.006	11.998	2.000
3	10.990	7.984	11.996	11.990	2.000
4	10.992	7.992	12.000	11.992	2.000
5	10.994	7.970	12.000	11.990	2.000
Average	10.992	7.982	12.000	11.990	2.000

Table 2 Measured mass of samples (unit: g)

Sample	Ring	Square bar	Round bar	Small ball
m	554.69	312.26	187.13	32.63

The rotational moments of inertia of various objects are calculated (unit: kg·m^2) as:

$$I_{ring} = \frac{1}{2}m\left(c^2 + b^2\right) = \frac{1}{2} \times 0.55469 \times \left(0.10992^2 + 0.07982^2\right) = 1.280 \times 10^{-3}$$

$$I_{square} = \frac{1}{12}md^2_{square} = \frac{1}{12} \times 0.31226 \times 0.12000^2 = 3.747 \times 10^{-4}$$

$$I_{round} = \frac{1}{12}md^2_{round} = \frac{1}{12} \times 0.18713 \times 0.11990^2 = 2.242 \times 10^{-4}$$

$$I_{ball} = 2 \times (mx^2 + \frac{2}{5}mR^2_{ball}) = 2 \times (0.03263 \times 0.05^2 + 0.4 \times 0.03263 \times 0.01^2) = 1.658 \times 10^{-4}$$

2. Measurement of rotational moments of inertia of steel and copper wires

Table 3 Shear modulus of steel wire (counts: 20; timing: 10 cycles)

R (mm)	L (cm)	Total T_0 (s)	Total T_0 (s)	$\overline{T_0}$ (s)	Total T_1 (s)	Total T_1 (s)	$\overline{T_1}$ (s)	G (N/m^2)
0.200	61.6	54.39	54.41	5.440	137.39	137.47	13.743	7.776×10^{10}
0.200	61.6	54.85	54.91	5.488	137.88	137.8	13.784	7.747×10^{10}
0.200	82.8	63.33	63.39	6.336	159.25	159.43	15.934	7.789×10^{10}
0.200	82.8	63.65	63.54	6.359	159.46	159.35	15.941	7.792×10^{10}

From Eq. (9), the shear modulus G of the steel wire is derived using row 1 in table 3:

$$G = \frac{8\pi L}{R^4} \times \frac{I_1}{T_1^2 - T_0^2} = \frac{8 \times 3.1416 \times 0.616}{(0.2 \times 10^{-3})^4} \times \frac{1.280 \times 10^{-3}}{13.743^2 - 5.440^2} = 7.776 \times 10^{10}\,\text{N/m}^2$$

By averaging the shear modulus in each row of table 3, we get:

$\overline{G}$=(7.776×10^{10}+7.747×10^{10}+7.789×10^{10}+7.792×10^{10})/4=7.776×10^{10} N/m^2

The recognized value of the shear modulus of the steel wire is 7.80×10^{10} N/m^2, thus the experimental error is pretty low.

Similarly, the shear modulus of the copper wire can be measured in comparison with the accepted value of 3.12×10^{10} N/m^2.

3. Measurement of rotational moments of inertia of square and round bars

Table 4 Rotational moments of inertia of square and round bars with steel wire (counts: 20; timing: 10 cycles)

	Total T_2 (s)	$\overline{T_2}$ (s)	Total T_0 (s)	$\overline{T_0}$ (s)	L_{wire} (cm)	I_2 (kg·m^2)		Error
						Theoretical	Experimental	
Round bar	88.21	8.821	63.39	6.339	82.5	2.242×10^{-4}	2.259×10^{-4}	0.8%
Square bar	101.58	10.158	63.33	6.333	82.5	3.747×10^{-4}	3.785×10^{-4}	1.0%

4. Measurement of moments of inertia of ball and ring using torsional coefficient of steel wire, and verification of perpendicular axis theorem.

Table 5 Verification of perpendicular axis theorem using steel wire (counts: 20, timing: 10 cycles)

No.	Total T_0 (s)	Total T_2 (s)	L (cm)	R (mm)	I_2 (kg·m^2)		Error
					Theoretical	Experimental	
1	63.65	121.82	82.8	0.200	6.400×10^{-4}	6.454×10^{-4}	0.8%
2	63.54	121.81					
Average	6.3595	12.1815					

As we know, the rotational moment of inertia of the ring rotating horizontally around central steel axis is I_1=1.280×10^{-3} kg·m^2. By comparison, the measured moment of inertia of the ring rotating vertically around the same steel wire is I_2=6.454×10^{-4} kg·m^2 as shown in Table 4. Thus, the ratio of I_1 to I_2 is 1.983 i.e. $I_1 \approx 2I_2$. The error is 0.8%. As a result, the perpendicular axis theorem is verified.