1. Theory

1) Hall Displacement Sensor

When a Hall element is placed in a magnetic field of magnetic induction strength B with current I applied in the direction perpendicular to the magnetic field, a Hall potential difference U_H will be generated by the Hall element in the direction perpendicular to B and I.

$$U_H = K \cdot I \cdot B \tag{1}$$

where K is the sensitivity of the Hall element. If current I remains unchanged while moving the Hall element in a magnetic field with a uniform magnetic field gradient, the variation amount of the Hall potential difference will be:

$$\Delta U_H = K \cdot I \cdot \frac{dB}{dZ} \cdot \Delta Z \tag{2}$$

where ΔZ is the displacement. Equation (2) means that when gradient dB/dZ is a constant, output ΔU_H is proportional to ΔZ.

To achieve a uniform gradient magnetic field, two pieces of magnets with same cross-section area and same surface magnetic flux density can be placed in opposite directions, e.g. N pole to N pole by keeping an equal spacing between the two poles, as seen in Figure 1. A Hall element is placed parallel to magnets in the middle axis of the gap whose width is determined by the required measurement range and measurement sensitivity. The smaller the gap is, the greater the magnetic field gradient and the hence higher sensitivity are. To reduce edge effect and thus improve measurement accuracy, the cross-sections of the magnets should be much greater than the dimensions of the Hall element.

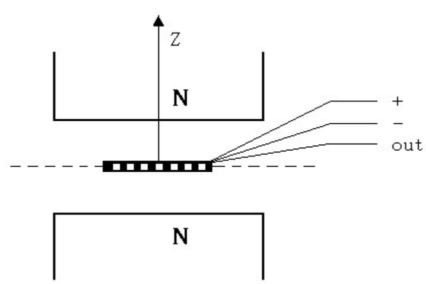


Figure 1 Schematic of the generation of Hall voltage

If the magnetic induction on the central cross-section of the magnet gap is zero, the output Hall potential difference of a Hall element placed in this location should be zero too. When the Hall element is moved away from the central section along Z-axis, a potential difference is created by the non-zero magnetic induction to the Hall element, which can be measured with a digital voltmeter. Correspondingly, the position with zero Hall potential difference can be set as the zero reference displacement.

There is a one-to-one relationship between Hall potential difference and displacement amount. When the displacement is relatively small (< 2 mm), this relationship is quite linear.

2) Young's modulus

As shown in the schematic of a Young's modulus apparatus in Figure 2, a horizontal beam (the sample) is bent by a vertical force, thus Young's modulus Y of the sample can be expressed as:

$$Y = \frac{d^3 \cdot Mg}{4a^3 \cdot b \cdot \Delta Z} \tag{3}$$

where d is the distance between two support edges, M is the weight of the applied weights, a and b are the thickness and the width of the sample sheet, respectively, ΔZ is the decline distance at the central point of the sample dragged by the weights, and g is the gravitational constant.

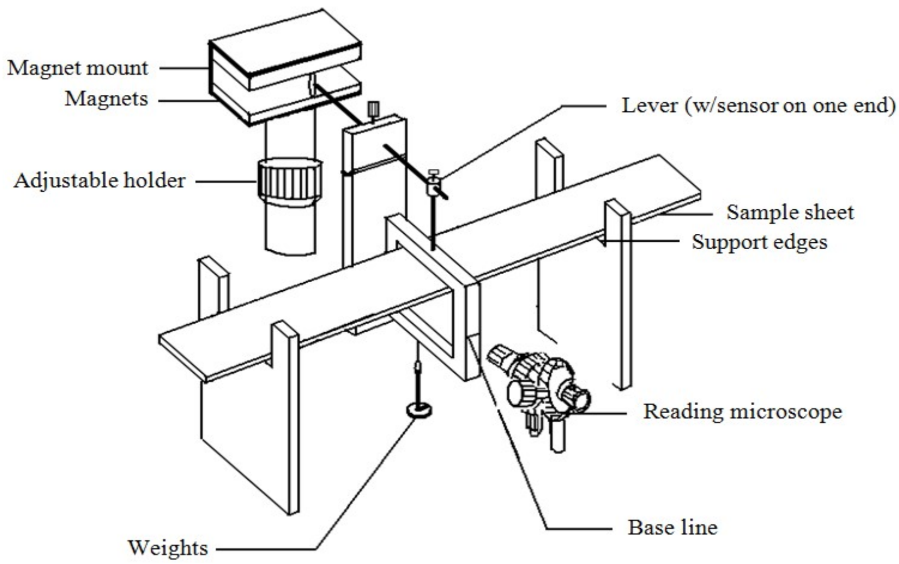


Figure 2 Schematic diagram of apparatus

Appendix: Derivation of formula for measuring Young's modulus by bending method

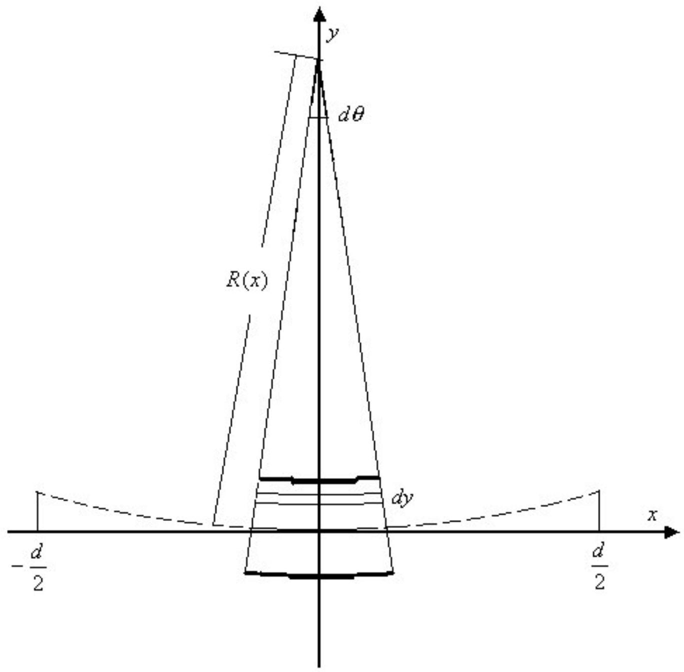
Under external force, the shape and volume of solid, liquid or gas are subject to change, called deformation. When external force is not too big, the resulted deformation will disappear after the withdrawal of external force. Such deformation is known as elastic deformation which can be categorized into three types, i.e. length, shear, and volume deformation.

When two opposite forces F of equal strength are imposed to either end of a solid rod of length l and cross section S, the rod is elongated by Δl. Within the elastic limit, according to Hooke's law, we have:

$$\frac{F}{S} = Y \cdot \frac{\Delta l}{l}$$

where F/S is the stress, $\Delta l/l$ is the relative length change (strain), and Y is the Young's modulus related to the property of the material.

In this experiment, when the beam is forced to bend, there exists a neutral section in the beam whose upper and lower sections are compressed and stretched, respectively. In general, it can be considered as a length change of the beam, which can described by the Young's modulus of the material. As shown in the figure below, the dashed line represents the neutral section of the beam, which is neither stretched nor compressed.



If we take a small segment dx of the bent beam by assuming its curvature radius is $R(x)$ and the corresponding open angle is $d\theta$, then the length of a layer with thickness dy above the neutral section with distance y is $(R(x)-y)\times d\theta$ after the beam bends. So the length is changed by:

$$(R(x)-y)\cdot d\theta - dx$$

Also,

$$d\theta = \frac{dx}{R(x)}$$

So we have:

$$(R(x)-y)\cdot d\theta - dx = (R(x)-y)\frac{dx}{R(x)} - dx = -\frac{y}{R(x)}dx$$

Therefore, the strain is:

$$\varepsilon = -\frac{y}{R(x)}$$

According to Hooke's law

$$\frac{dF}{dS} = -Y\frac{y}{R(x)}$$

Also

$$dS = b \cdot dy$$

So we have:

$$dF(x) = -\frac{Y \cdot b \cdot y}{R(x)} dy$$

The torque relative to the neutral section is:

$$d\mu(x) = |dF| \cdot y = \frac{Y \cdot b}{R(x)} y^2 \cdot dy$$

By integral operation, we get:

$$\mu(x) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{Y \cdot b}{R(x)} y^2 \cdot dy = \frac{Y \cdot b \cdot a^3}{12 \cdot R(x)} \tag{1}$$

For the points on the beam, we have:

$$\frac{1}{R(x)} = \frac{y''(x)}{\left[1 + y'(x)^2\right]^{\frac{3}{2}}}$$

Since the bending is tiny, $y'(x) = 0$, therefore:

$$R(x) = \frac{1}{y''(x)} \tag{2}$$

When the beam is in balance, the beam torque at point *x* should be equal to the moment of the support force Mg/2 at the knife edge relative to point *x*, i.e.

$$\mu(x) = \frac{Mg}{2}(\frac{d}{2} - x) \tag{3}$$

From Eq. (1), (2), and (3), we have:

$$y''(x) = \frac{6Mg}{Y \cdot b \cdot a^3}(\frac{d}{2} - x)$$

Considering boundary conditions: $y(0)=0$ and $y'(0)=0$, we get:

$$y(x) = \frac{3Mg}{Y \cdot b \cdot a^3}(\frac{d}{2}x^2 - \frac{1}{3}x^3)$$

By substituting x=d/2 into the above expression, we get y value at right endpoint (knife edge)

$$y = \frac{Mg \cdot d^3}{4Y \cdot b \cdot a^3}$$

Also, $y=\Delta Z$. Therefore, the Young's modulus is derived as:

$$Y = \frac{d^3 \cdot Mg}{4a^3 \cdot b \cdot \Delta Z}.$$

2. Apparatus Structure and Specifications

1) This apparatus includes one electronic unit and one mechanical unit. The former consists of a Hall sensor signal measuring device and a digital DC voltmeter, while the latter consists of a base box, a reading microscope, a Hall sensor, magnets, and other parts.
2) Key specifications:
   a) Reading microscope: range: 6 mm; resolution: 0.01 mm; magnification: 20 ×
   b) Weights: 10.0 g and 20.0 g
   c) Digital voltmeter: 3-1/2 digit; range: 0 ~ 2000 mV
   d) Samples: brass and malleable cast-iron sheets
   e) Relative uncertainty of measurement: < 3%
   f) Hall displacement sensor: sensitivity > 250 mV/mm; linear range 0-2 mm

3. Apparatus Setup

1) Open the package, place the mechanical box on a stable platform with the side of screws facing up, unscrew the 4 fixing screws of the board which are located at the four corners of the board (do not confuse with the 4 base feet screws). Tilt the box and take out the board carefully (note: there are some parts mounted on the other side of the board). Place the board aside and let the side with parts facing up.
2) Mount the magnets and the reading microscope with corresponding holders on the board as shown in Figure 2. Place the samples (both brass sheet and iron sheet) on the experimental table. Check the remaining parts in the bag, including: 8 pieces of 10.0-g weight, 2 pieces of 20.0-g weight, one copper lever assembly (including level rod, integrated Hall sensor, knife edge, cylinder, three-hole plug with leads), one copper rectangular frame with baseline, one weight holder, and 3 screws for leveling the base.
3) Mount the three leveling screws to the bottom of the base box. Place the board on the top of the base box, and restore the 4 fixing screws to tighten the board. Place the level bulb on the board and adjust the leveling screws to level the board.
4) Insert the beam (one of the sample sheets) in the rectangular copper frame and place it onto the central position of the two support edges. Then mount the copper lever assembly by placing the end with Hall sensor in the middle of the two magnets, the supporting knife edge in the V-groove of the middle post, and the pin tip at the bottom of the cylinder into the small pit on the top side of the copper frame. If the senor is not in the middle of the magnet gap, the height of the magnet can be adjusted by either loosening the fixing screw of the magnet post or rotating the height adjustment sleeve nut on the support post. **Note**: the Hall sensor should be in a horizontal plane (there are two fixing screws on the lever assembly, which should be loosened before rotating the lever rod to adjust the sensor plane direction).
5) Insert the 3-wire plug on the lever assembly into the 3-pin mount on the post, connect the electric unit with the other 3-pin mount using the cable provided; turn on apparatus power, adjust the magnet height or/and the "Zero" knob on the electric unit to zero apparatus reading at the original status. Preheat the apparatus for 10 minutes.

6) Adjust the eyepiece of the reading microscope until clear crosshair, reticle and numbers are observed. Adjust the height of the microscope up and down until a clear baseline on the frame is observed through the microscope (to adjust the height of the microscope, hold the microscope, loosen the top locking nut and the side fixing screw, rotate the large nut until a proper height is achieved, then tighten the side screw and fix the locking nut.) Turn the reading knob of the microscope to bring the crosshair to overlap with the baseline.

4. Precautions

1) The thickness of the beam (sample) must be measured accurately with a caliper.
2) The crosshair of the reading microscope should be aimed at the baseline on the frame rather than the frame edge.
3) Before calibrating the Hall sensor, zero the output of the Hall sensor by turning the adjustment screw of the support post. Further, the sensor probe should be placed in the middle or slightly lower position of the magnet gap.
4) Add or remove weights gently to minimize the swing of the frame.
5) Check whether the beam is bent before experiment. If so, correct it.

5. Experimental Contents

1) Understand the characteristics of a Hall displacement sensor
2) Measure the Young's modulus of a brass sample using the bending method
3) Calibrate a Hall displacement sensor
4) Measure the Young's modulus of a malleable iron sample

6. Experimental Procedure

A. Measure the Young's modulus of a brass sample and calibrate a Hall displacement sensor

1) Adjust the height of the Hall sensor to let it locate in the middle of the magnet gap.
2) Check if the base is leveled horizontally. If not, adjust base screws to level the base.
3) Zero the voltmeter. When the reading is relatively large, adjust the magnet height by turning the adjustment screw under the magnet box; when the reading is small, adjust the "Zero" knob on the electric unit.
4) Adjust the eyepiece of the reading microscope until clear crosshair, reticle and numbers are observed. Adjust the reading microscope height until a clear baseline on the frame is observed through the microscope. Turn the reading knob of the microscope to bring the crosshair to overlap with the baseline. Record the microscope reading.
5) Sequentially add weight M_i (10 g at a time) while recording the corresponding bending displacement ΔZ_i of the beam measured with the reading microscope and the sensor output voltage U_i (mV) measured with the digital voltmeter.
6) Measure distance d between the two support edges of the beam, and beam thickness a and width b at different locations.
7) Calculate the Young's modulus of the brass sample using Eq. (3), and derive sensitivity $\Delta U_i/\Delta Zi$ of the Hall displacement sensor.

B. Optional experiment: measure the Young's modulus of malleable cast iron.

7. Examples of Data Recording and Processing

Note: Following data are for reference purpose only, not the criteria for apparatus performance:

1) Calibration of a Hall displacement sensor

   Prior to measurement, check whether the lever is leveled, the knife-edge is vertical, the beam is centered, and the sensor is in the middle of magnet gap without contacting the metal shell. Then, add weights to create bending displacement ΔZ for the beam while measuring output U of the sensor and position Z of frame baseline. Measurement data are shown in Table 1, showing a good linear relationship between U and Z.

Table 1 Static characteristics measurement of a Hall displacement sensor

M (g)	0	20	40	60	80	100
Z (mm)	0	0.24	0.49	0.73	0.99	1.24
U (mV)	0	73	145	216	290	365

2) Measurement of Young's modulus

   Measure distance d between the two edges of the beam, measure beam width b and beam thickness a using a caliper. The measurement data are: d=23.00 cm, b=2.30 cm, and a=0.995 mm, respectively. Use the above data, measure beam displacement under different weight load. Data are shown in Table 2.

Table 2 Displacement of brass sample under different weight load

M (g)	0	20	40	60	80	100
Z (mm)	0.00	0.240	0.490	0.730	0.990	1.240

Using the data in Table 2, calculate displacement ΔZ at weight M=60 g.

For example, if the voltage is measured at 216 mV with a 60-g weight, and the beam bending displacement is measured ΔZ=0.730 mm. By substituting these data to Eq. (3), we get the Young's modulus of the brass sample as:

$$Y_{brass} = \frac{(23.00\times10^{-2})^3\times60.00\times10^{-3}\times9.794}{4\times(0.995\times10^{-3})^3\times2.30\times10^{-2}\times0.730\times10^{-3}} = 10.65\times10^{10}\,N/m^2$$

The sensitivity of the Hall displacement sensor is:

$$K_{brass} = \frac{\Delta U}{\Delta Z} = \frac{216}{0.73} = 2.96\times10^2\,mV/mm$$

By comparison, a reference Young's modulus value of brass material is Y_0=10.55×10^{10} N/m^2. Thus, the measurement error is about 1%.

Similarly, the experimental results for a cast iron sample are:

$$\Delta Z_{\mathrm{Fe}} = \frac{U}{K} = 0.44mm$$

$$Y_{\mathrm{Fe}} = \frac{(23.00\times10^{-2})^3\times60.00\times10^{-3}\times9.794}{4\times(1.001\times10^{-3})^3\times2.30\times10^{-2}\times0.440\times10^{-3}} = 18.51\times10^{10}\,N/m^2$$

A reference Young's modulus value of cast iron material is Y_0=18.15×10^{10} N/m^2. Thus, the measurement error is about 2%.