## 2. Theory

### A. Nonlinear circuits and nonlinear dynamics

As shown in Figure 1, only one nonlinear element, an active nonlinear negative resistor $R$, is used in the circuit schematic of the experiment. Inductor $L$ and capacitor $C_2$ form a resonant circuit with negligible loss while variable resistor $R_0$ and capacitor $C_1$ connecting in series shift the phase of the sinusoidal signal generated by the oscillator. The nonlinear component $R$ is a five-piecewise linear element. Figure 2 show the volt-ampere (V-I) characteristic curve of this resistor. It can be seen from Figure 2 that the polarities between voltage and current applied to this non-linear component are opposite. An increase in the voltage applied to this component results in a decrease in the current, known as nonlinear negative resistance component. Figure 3 shows the schematic of the nonlinear component.

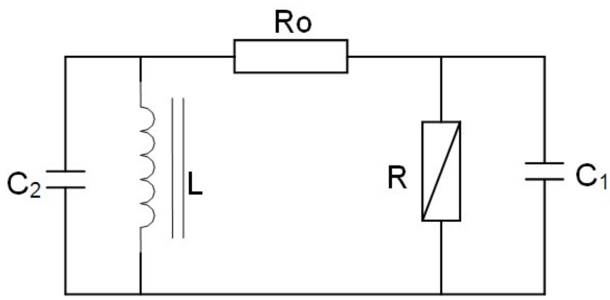


Figure 1 Schematic of a nonlinear circuit

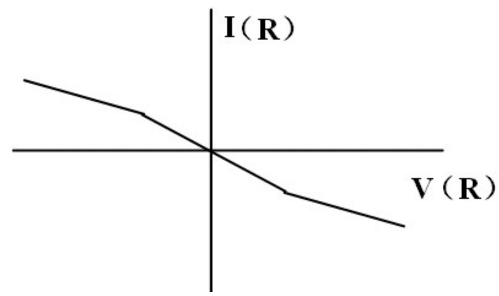


Figure 2 V-I characteristic of a nonlinear circuit

The nonlinear dynamics equations of Figure 1 are as follows:

$$
\begin{aligned}
C_1 \frac{dU_{C1}}{dt} &= G(U_{C2} - U_{C1}) - gU_{c1} \\
C_2 \frac{dU_{C2}}{dt} &= G(U_{C1} - U_{C2}) + i_L \\
L \frac{di_L}{dt} &= -U_{C2}
\end{aligned}
\tag{1}
$$

Where $U_{C1}$ and $U_{C2}$ are the voltages across $C_1$ and $C_2$, respectively; $i_L$ is the current in inductor $L$; and $g=1/R_0$ is the conductivity. Here $g$ is a function of $U$. If $R$ is linear, $g$ will be a constant. Under this case, the circuit is a general oscillation circuit and its solution is a sine function. The role of resistor $R_0$ is to adjust the phase difference between $C_1$ and $C_2$. When viewing on an oscilloscope, $U_{C1}$ and $U_{C2}$ should form an oval.

Since $g$ is generally a non-linear function, the ternary nonlinear equations (1) do not have analytical solutions. By using numerical calculation through computer simulation with appropriately selected circuit parameters, chaos phenomena of the simulation experiments can be observed. Beside computer numerical simulation method, a more direct method is to use an oscilloscope to observe the chaos created by circuits shown in Figure 3. The key component in the circuit is the nonlinear resistor through the combination of a dual operational amplifier and six resistors. In the circuit, $L$ and $C$ connecting in parallel form an oscillation circuit. The role of $R_0$ is for phase shifting to create a phase difference at the two output terminals $J_1$ and $J_2$ that are connected to X and Y of an oscilloscope.

Both positive and negative feedbacks of the front and back stages of the dual operational amplifier LF353 are simultaneous. The gain of positive feedback is related to the ratio of $R_3/R_0$ to $R_6/R_0$, as the gain of negative feedback is associated with the ratio of $R_2/R_1$ to $R_5/R_4$. When positive feedback is larger than negative feedback, the oscillation circuit maintains oscillation. By adjusting $R_0$, positive feedback changes and LF353 remains oscillation, thus demonstrating nonlinear characteristic. Seen from points *C* and *D*, the combination of one LF353 and six resistors is equivalent to a nonlinear resistor.

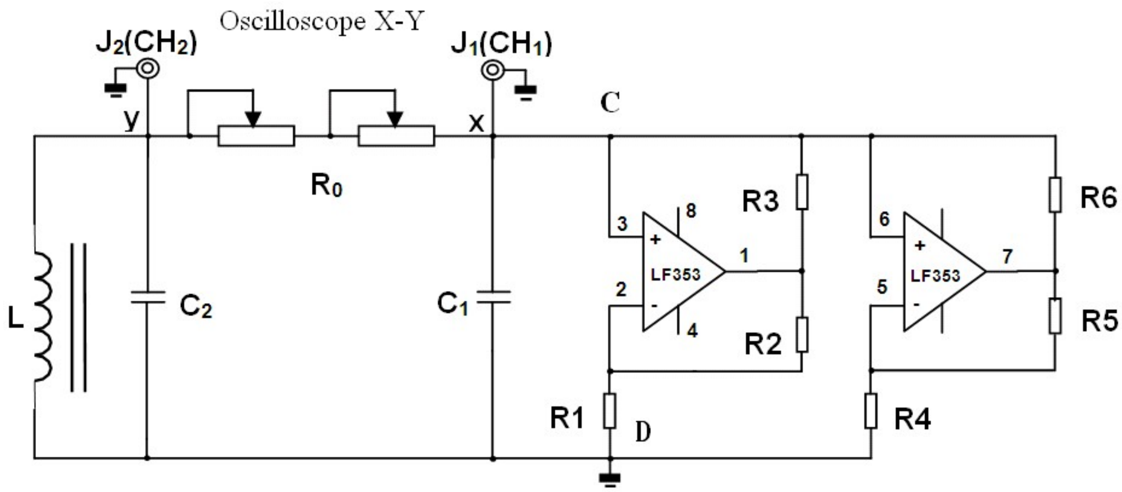


Figure 3 Schematic of experimental circuit

**B. Implementation of active nonlinear negative resistance device**

There are many methods for the implementation of active nonlinear negative resistance devices. This apparatus uses two operational amplifiers (a dual Op-Amp LF353) and six resistors. Its circuit is shown in Figure 3. The configuring resistors in the symmetric dual Op-Amp differ by 100 times in resistance, making the total output current have different characteristics in different working voltage regions. For example, one amplifier reaches saturation but the other does not, resulting in a nonlinear current-voltage characteristic. Its volt-ampere characteristic curve is shown in Figure 2. This experiment is to study the effect of a nonlinear component on the entire circuit. The role of a nonlinear negative resistance component is to form nonlinear phenomena on oscillation period, such as bifurcation and chaos. The nonlinear chaotic circuit of the apparatus is shown in Figure 3.

**C. V-I characteristic of active nonlinear resistor**

The experimental circuit is shown in Figure 4, in which *R*' is an active nonlinear negative resistance component (dual Op-Amp) and *R* is an external resistor.

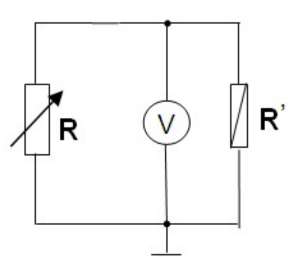

Figure 4 Schematic of experimental circuit

Active nonlinear negative resistance elements generally satisfy the characteristic curve of the so-called “Chua's circuit”. In this experiment, the LC oscillation circuit is separated from the nonlinear resistor. The voltmeter is used to measure the voltage between the two ends of the nonlinear component. Because the nonlinear resistor is active, there is always current in the circuit. Here $R$ is a resistance box, used to change the output of the non-linear component. A resistance box can change resistance very precisely, so as to change the output finely.