4. Operation of Apparatus

1) Open the cover of the main unit case, and connect the ferrite inductor to corresponding terminals on the panel (pre-mounted prior to delivery).
2) Connect CH1 and CH2 ports on the panel to X and Y axes of an oscilloscope using BNC cables. Set X and Y input mode of the oscilloscope at "DC" in order to display the Lissajous figure of two sine waves (phase figure).
3) Turn on the power of the main unit, the power indicator and digital voltmeter should be on.
4) If the digits of the voltmeter flash, it means the input voltage exceeds the upper limit.

5. Precautions

1) The positive and negative polarities of the dual Op-Amp must be connected correctly; the ground terminal "GND" of the main unit must be connected to the ground of the lab power source.
2) Power must be turned off prior to connecting or disconnecting wires on the panel.
3) Warm-up 10 minutes before taking measurement data.

6. Experimental Contents

1) Use RLC series resonance circuit to measure the inductance of a ferrite material at different currents;
2) Observe the waveforms generated by a LC oscillator on an oscilloscope before and after RC phase-shift, respectively (oscilloscope in X-T mode);
3) Observe the phase figure of the above two waveforms (i.e. Lissajous figure, oscilloscope in X-Y mode));
4) Observe the periodic variations of the phase figure by adjusting the resistors of the RC phase shifter; Record phase figures of bifurcations, intermittency chaos, triple times period, attractor, and double attractors;
5) Measure the V-I characteristics of a nonlinear negative resistance device made of a LF353 dual operational amplifier; Explain the cause of the chaos generation using the dynamics equation of a nonlinear circuit.

7. Experiments

A. Observation of period-doubling bifurcation and chaos in nonlinear circuits

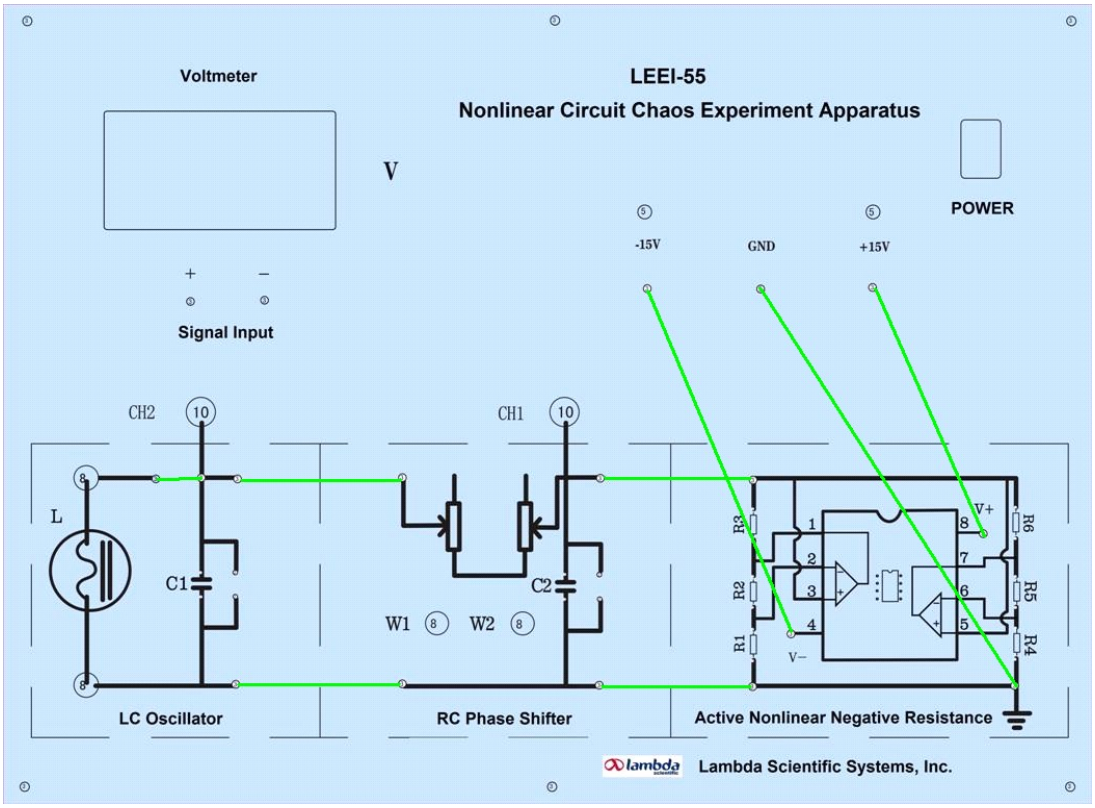

Figure 5 Wiring schematic of experiment A

a) Connect wires on the panel per Figure 5 and connect CH1 and CH2 to X and Y of the oscilloscope, respectively;

b) R_0 consists of two variable resistors in serial, W_1 and W_2. W_1 is for coarse adjustment while W_2 for fine adjustment. Adjust W_1 and W_2 (R_0=W_1+W_2) while observing Lissajous figure (phase figure) on oscilloscope;

c) Adjust R_0 from large to small while depicting period bifurcation and chaos of the phase figure; (Note: turn in count-clockwise direction to decease resistance.)

d) Using P as the period of a ring phase figure, observe and record phase figures: 2P, 4P, intermittent chaos, 3P, single-attractor (chaos), and double attractors (chaos).

B. Measurement of V-I characteristic of active nonlinear negative resistance element

a) Connect circuit per Figure 6;

b) Gradually increase resistance of the resistance box from small (e.g. 500 Ω) to large (e.g. 500K Ω), record voltage readings from the voltmeter at various resistances (note: V<0);

c) Acquire current-voltage characteristics of the nonlinear circuit using the recorded data;

d) Plot *I-V* graph with least square curve fitting;

e) Understand the concept of negative resistance from the *I-V* curve.

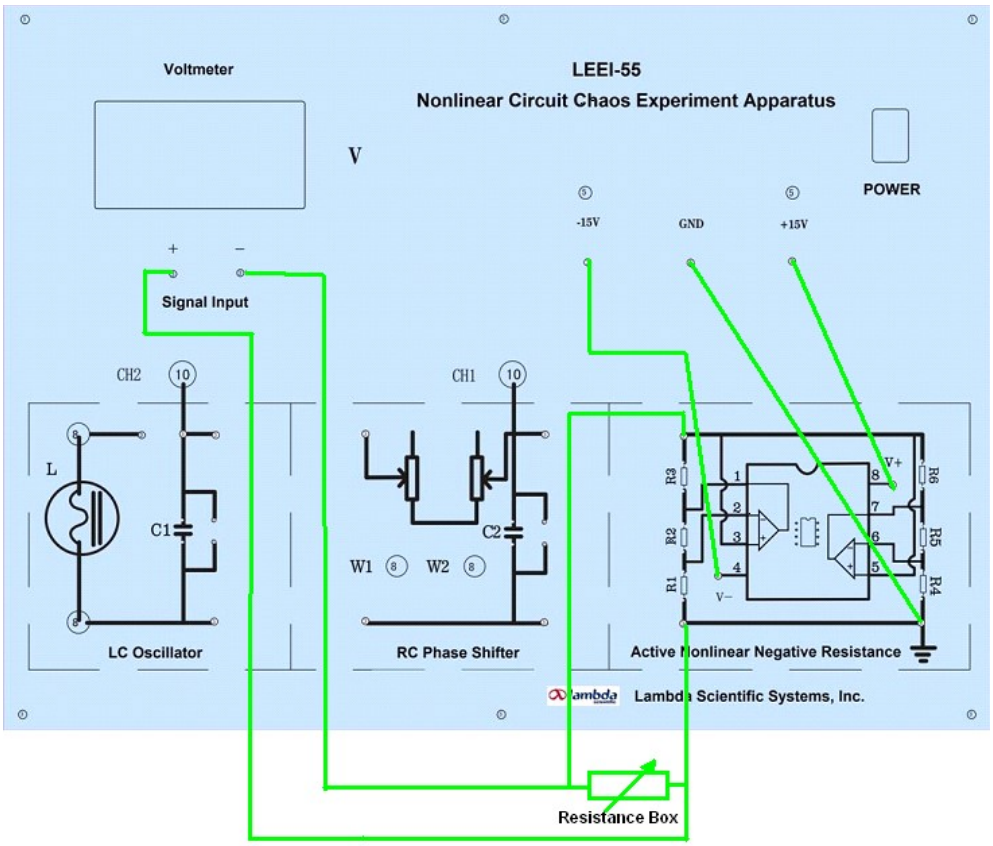

Figure 6 Wiring schematic of experiment B

C. Measurement of the inductance of a Ferrite inductor

1) There are two options for selecting inductor L: i) use the built-in ferrite inductor or ii) user makes the inductor using included ferrite core and frame by winding 70-75 turns of copper wire, and scrape off paints of the two leads for good contact.
2) Measure the inductance of the inductor using series resonance method (see Fig. 8). Connect the inductor and resistance box (set at 30 Ω) in series, then connect a low frequency signal generator. Measure the voltage between the two ends of the resistance box using the oscilloscope and adjust the frequency of the sine wave signal to achieve the maximum voltage value on the resistance box, while measuring the current, I, flowing through the resistance box. Measure the inductance of the inductor when the current reaches 5 mA (effective value).

8. Example of Data Recording and Processing

Note: The following data are for reference only, not the criteria for apparatus performance:

A. Observation of period-doubling bifurcation and chaos

1) Refer to Figure 5 to connect circuit. Set R_0 (i.e. $1/g$ or W_1+W_2) at a certain large value, now, Lissajous figure should appear on the oscilloscope (with X-Y mode), as shown in Figure 7 (a). Use X-t and Y-t scanning mode for the oscilloscope, two sine waveforms with some phase shift (phase difference) can be observed.

2) Gradually reduce 1/*g* value, and two split rings should emerge, known as the bifurcation phenomenon, that is, the original one time period becomes two time period, as shown in the Lissajous figure in Figure 7 (b).
3) Continuously reduce 1/*g* value, 4 times as shown in Figure 7 (c), 8 times, and 16 times periods, and alternative intermittency chaos phenomena should all appear. Intermittency chaos is shown in Figure 7 (d).
4) Reduce 1/*g* further, 3 times period should appear, as shown in Figure 7 (e). According to Yorke's famous assertion, "3 times period means chaos," indicating that chaos will occur in the circuit soon.
5) Continuously reduce 1/*g*, single attractor occurs, as shown in Figure 7 (f).
6) Continuously reduce 1/*g*, double attractors occur, as shown in Figure 7 (g).

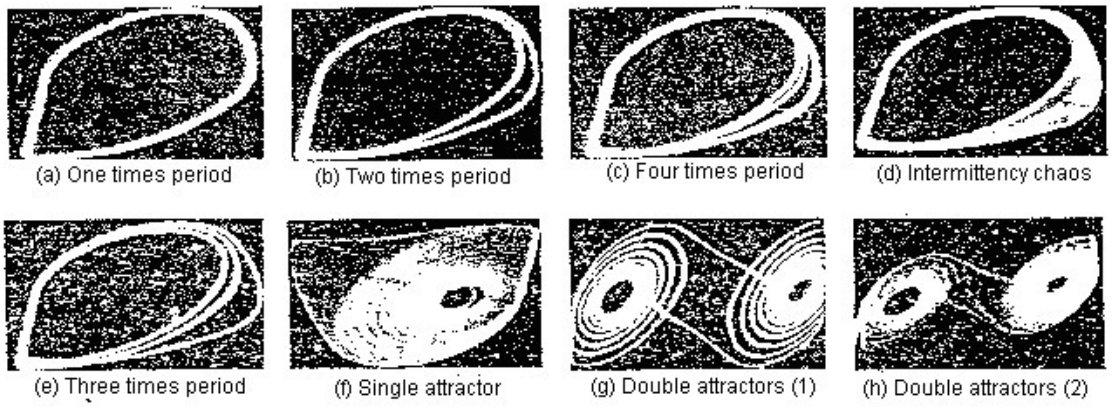

Figure 7 Serial figures of multi-period bifurcation

B. Measurement of V-I characteristic of active nonlinear resistor

Table 1 V-I characteristic of active nonlinear resistor

U (V)	R (Ω)	I (mA)	U (V)	R (Ω)	I (mA)
-0.010	600.0	0.017	-6.000	1944.7	3.085
-0.100	1162	0.086	-7.004	2042.7	3.918
-0.200	1230.2	0.163	-9.004	2077.3	4.334
-0.400	1267.7	0.316	-10.007	2105.6	4.753
-0.601	1280.1	0.469	-10.801	2124.5	5.084
-0.802	1287.3	0.623	-11.002	2178.9	5.168
-1.004	1291.3	0.778	-11.202	2148.0	5.215
-1.196	1293.8	0.924	-11.401	2400.0	4.750
-1.395	1295.7	1.077	-11.601	2840.0	4.085
-1.600	1297.1	1.234	-11.802	3500.0	3.372
-1.800	1344.4	1.339	-12.001	4550.0	2.638
-2.000	1405.4	1.423	-12.403	11000.0	1.128
-4.000	1774.0	2.255	-12.600	32000.0	0.394
-5.000	1872.7	2.670			

By dividing the data in Table 1 into 3 segments and conducting linear curve-fitting for each segment based on a linear equation $I = AV + B$, the following parameters are achieved:

For U from -0.200 V to -1.600 V, A_1= -7.649×10^{-4} A/V, B_1=9.547×10^{-3} mA, r=0.99999.
For U from -2.000 V to -10.007 V, A_2= -4.157×10^{-4} A/V, B_2=0.592 mA, r=0.99999.
For U from -11.202 V to -12.600 V, A_3= -3.526×10^{-3} A/V, B_3=44.90 mA, r=0.9985.

Here, A represents the slope of a straight line, B is the intercept, and r is correlation coefficient. The intersections of the three lines are as follows: U_1 =-1.6680 V, I_1 =1.285 mA; and U_2=-11.241 V, I_2=5.265 mA.

C. Measurement of inductance

The circuit for measuring inductance is shown in Figure 8. First, measure the voltage across R using CH2 while keeping the output voltage of the signal generator unchanged. Adjust the frequency of the signal generator until the measured voltage on CH2 reaches maximum. Now, RLC series circuit achieves resonance condition. Thus, we have:

$$\varpi L = 1/\varpi C,\ f_0 = 1/2\pi \cdot \sqrt{LC}$$

where f_0 is the resonance frequency. So,

$$L=1/4\pi^2 C f_0^2,\ U_R=U_{CH2}/2\sqrt{2}$$

where U_{CH2} is the peak-peak voltage of CH2 and U_R is the voltage across R. Thus, the effective value of current in circuit is

$$I=U_R/R$$

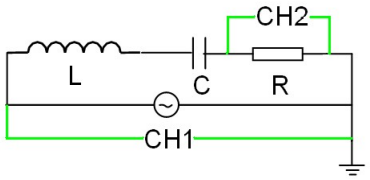

Figure 8 Circuit for measuring inductance

Measurement data of two inductance samples are provided below. They are only for reference purpose as the data are completely different for different inductors. However, the measurement method and data recording are similar.

Sample A

Table 2 Data of inductance L versus current I

f_0 (kHz)	I (mA)	L (mH)
3.14	19.7	25.7
3.19	16.0	24.9
3.23	12.2	24.3
3.30	8.29	23.3
3.39	4.26	22.0
3.44	2.16	21.4
3.47	1.74	21.0
3.49	1.10	20.8

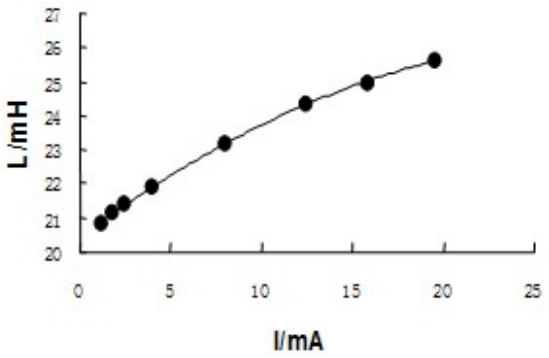

Figure 9 Inductance *L* versus current *I* of sample A

It can be seen from the data in Table 2 that inductance, *L*, increases with an increase of current *I*, meaning that there is a magnetic core in the inductor.

Sample B

Table 3 Measurement data of inductance parameters

Parameter	Value
R (Ω)	100.0
U_{CH2} (V)	12.0
I (mA)	42.4
f_0 (kHz)	1.995
L (mH)	29.6
R_L (Ω)	33.3

Table 4 Data after changing voltage of signal generator

Parameter	Value
R (Ω)	100.0
U_{CH2} (V)	4.00
I (mA)	14.1
f_0 (kHz)	2.038
L (mH)	28.3

Table 5 Inductance *L* versus current *I*

U_{CH2} (V)	I (mA)	f_0 (kHz)	L (mH)
12.0	42.4	1.995	29.6
10.0	35.5	1.980	30.1
8.00	28.3	1.982	30.0
6.00	21.2	2.000	29.5
4.00	14.1	2.038	28.3
2.00	7.07	2.110	26.5

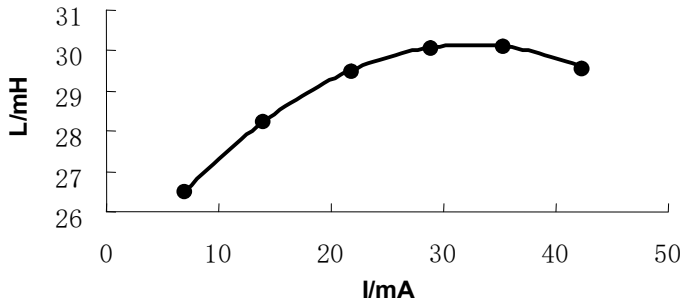

Figure 10 Inductance *L* versus current *I* of sample B

Similarly, inductance *L* increases with an increase of current *I*, meaning there exists a magnetic core in the inductor. The inductance reaches saturation when the current exceeds 25 mA, over which the inductance gradually decreases. This is because magnetic permeability μ falls when the current in the inductor becomes greater, so the inductance will be reduced.