2. Theory

2.1 Measurement of PN junction current-voltage characteristics and Boltzmann constant

From semiconductor physics, the forward current through a PN junction can be expressed as:

$$I = I_0\left[e^{\frac{eU}{kT}} - 1\right] \tag{1}$$

where I_0 is the reverse saturation current, which is a constant under a constant temperature; T is the thermodynamic temperature; e is the charge of the electron, and U is the forward voltage drop of the PN junction. At room temperature (T=300 K), $kT/e \cong 0.026$ V, while the forward voltage drop of PN junction is about a few tenths volt, we have exp (eU/kT)>>1. Thus, Eq. (1) can be simplified as:

$$I = I_0 e^{\frac{eU}{kT}} \tag{2}$$

This means that the forward current of a PN junction is an exponential function of the forward voltage. If the *I-U* relationship of a PN junction is measured, e/kT can be derived from Eq. (1). After temperature T is measured, e/k can be known, from which the Boltzmann constant k can be further derived.

In practice, although the *I-U* relationship of a diode satisfies the exponential law well, constant k derived from the *I-U* relationship is often smaller than the recognized value. This is because the current flowing through the diode is not just composed of the diffusion current, but other components as listed below:

1) Diffusion current: strictly follows Eq. (2);
2) Accord current in depletion layer: is proportional to $e^{eU/2kT}$;
3) Surface current: is proportional to $e^{eU/mkT}$ (usually $m > 2$).

To minimize or eliminate the effect of unwanted current components on the derivation of ratio e to k using Eq. (2), a common-base silicon transistor rather than a silicon diode should be used. Under such case, the collector and the base of the transistor are connected, so only diffusion current exists in the collector current.

This apparatus uses a silicon transistor (i.e. TIP31) under a low positive bias, so the influence of surface current can be neglected. The collector current and junction voltage satisfy Eq. (2) very well. A schematic diagram of the experimental circuit is shown in Figure 1.

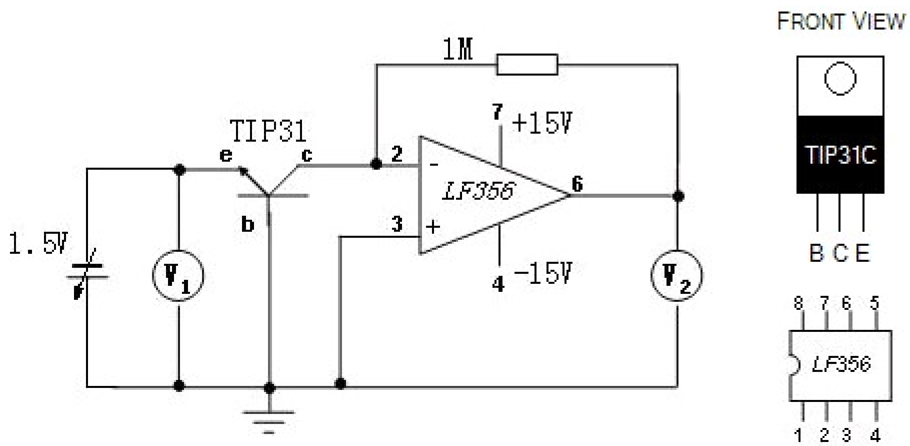

Figure 1 Circuit for measuring diffusion current and junction voltage of PN junction

2.2 Measurement of weak current

In the past, weak current in range of 10^{-6}A to 10^{-11} A was usually measured using a reflective spot type galvanometer with sensitivity as high as 10^{-9} A/division, however, such conventional galvanometer is neither robust nor user friendly as it is extremely sensitive to vibration in the environment, requiring frequent maintenance and repair. Hence, the conventional galvanometer is replaced by the operational amplifier (op-amp) type current-voltage converter for the measurement of weak current, yielding a number of attractions such as low input impedance, high current sensitivity, small temperature drift, good linearity and simple circuit design. The schematic diagram of a current-voltage converter using Op-Amp LF356 is shown in Figure 2.

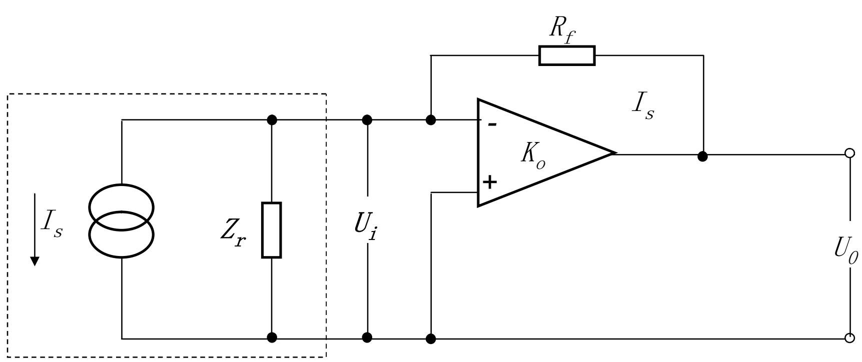

Figure 2 Schematic of current-voltage converter using op-amp LF356

As shown in Figure 2, resistance Z_r in the dotted frame is the equivalent input impedance of the current-voltage converter. The output voltage of the op-amp, U_0, is:

$$U_0 = -K_0 U_i \quad (3)$$

where U_i is the input voltage, K_0 is the open-loop voltage gain of the operational amplifier, and R_f is the feedback resistor. As the input impedance of an ideal operational amplifier is infinite ($r_i \to \infty$), the input current from the signal source flows through the feedback pathway only

$$I_S = (U_i - U_0)/R_r = U_i(1+K_0)/R_f \quad (4)$$

From (4), the equivalent input impedance, Z_r, of the current-voltage converter can be derived as

$$Z_r = U_i / I_s = R_f /(1 + K_0) \approx R_f / K_0 \tag{5}$$

From (3) and (4), the relationship between input current I_s and output voltage U_0 of the current-voltage converter can be derived as

$$I_s = -\frac{U_0}{K}(1 + K_0) / R_f = -U_0(1 + 1/K_0) / R_f = -U_0 / R_f \tag{6}$$

By measuring output voltage U_0 while using the known value of feedback resistor R_f, input current I_s can be found. For example, the open-loop gain and input impedance of an operational amplifier (LF356) are $K_0=2\times10^5$ and $r_i=10^{12}$ Ω, respectively. If R_f is set at 1.00 MΩ, from Eq. (5), we have $Z_r=1.00\times10^6$ Ω/$(1+2\times10^5)$=5 Ω.

If a 4-1/2-digit voltmeter with a range of 200 mV is used, the minimum change of its last digit is 0.01 mV. Then, the minimum current value displayed by the above current-voltage converter is: $(I_s)_{min}=0.01\times10^{-3}$ V/(1×10^6 Ω)=1×10^{-11} A.

2.3 Measurement of relationship between junction voltage and temperature

From semiconductor physics, when weak constant current on the order of 1 mA flows through a PN junction, junction voltage U_{be} and temperature T can be approximated as:

$$U_{be} = ST + U_{go} = ST + \frac{E_{go}}{e} \tag{7}$$

where S is the sensitivity of the PN junction temperature sensor ($S\cong$-2.3 mV/°C), and E_{go} is the prohibited bandwidth of the semiconductor material at temperature T=0 K. For silicon, E_{go} is about 1.20 eV.