2. Theory

Under certain conditions, the resistance of a conductive material varies with the magnetic field intensity *B* applied. This phenomenon is called the magnetoresistive effect. As seen in Figure 1, when a semiconductor is placed in a magnetic field, its carriers are subject to deflection driven by the Lorentz force. Thus, charges accumulate at both sides to create a Hall electric field. If the electric force and Lorentz force cancels out each other for a carrier of a specific speed, then other carriers with a different speed are still deflected. An end result of this process is that the number of carriers moving along the applied electric field reduces; in other words, resistance increases, showing a transversal magneto-resistance effect.



Figure 1 Schematic of magnetoresistive effect

As shown in Figure 1, if sides *a* and *b* of the semiconductor element are short-circuited, the magnetoresistive effect will be more obvious. Usually, a relative change of resistivity is used to represent the quantity of the magneto-resistance, i.e. $\Delta \rho / \rho_0$, where ρ_0 is the resistivity of the sensor in the absence of a magnetic field. In the presence of a magnetic field *B*, the resistivity of the sensor is ρ_B , hence we have $\Delta \rho = \rho_B - \rho_0$. Alternatively, the quantity of magnetoresistive effect can also be represented by the relative change of resistance ($\Delta R/R_0$) of the sensor.

In a weak magnetic field, the relative change of resistance $(\Delta R/R_0)$ of a magnetoresistive sensor is proportional to the square of the magnetic field intensity *B*; in a strong magnetic field, the relative change $(\Delta R/R_0)$ is linear with the magnetic field *B*. This property of a magnetoresistive sensor has important applications in physics and electronics.

If a magnetoresistive sensor is in a weak magnetic field modulated sinusoidally with frequency ϖ , its resistance R_B changes periodically at frequency 2ϖ since the magnetic resistance change $(\Delta R/R_0)$ is proportional to the square of the weak magnetic field intensity. In other words, a magnetoresistive sensor has a frequency-doubling characteristic in a weak AC magnetic field.

If the external AC magnetic field intensity is *B*, we get

$$B = B_0 \cos(\varpi t) \tag{1}$$

where B_0 is the amplitude of the magnetic field intensity, ϖ is angular frequency, and t is time. In a weak magnetic field, we have:

$$\Delta R / R_0 = KB^2 \tag{2}$$

where K is a constant. From (1) and (2), we have:

$$R_{B} = R_{0} + \Delta R = R_{0} + R_{0}KB^{2} = R_{0} + R_{0}KB^{2}_{0}\cos^{2}(\varpi t)$$

= $R_{0} + \frac{1}{2}R_{0}KB^{2}_{0} + \frac{1}{2}R_{0}KB^{2}_{0}\cos(2\varpi t)$ (3)

where the sum of the first two terms is a resistance that does not change with time, but the third term changes with time at angular frequency 2ϖ . Thus, the resistance of a magnetoresistive sensor in a weak sine-wave AC magnetic field will produce a frequency doubling effect.