2. Theory

Based on the Biot-Savart law, the magnetic field intensity $B$ at a specific point on the axis of a current-carrying coil (through the center and perpendicular to the plane of the coils) is:

$$ B = \frac{\mu_0 \cdot R^2}{2(\bar{R}^2 + x^2)^{3/2}} \cdot N \cdot I \quad (1) $$

where $\mu_0$ is the permeability in vacuum, $\bar{R}$ is the average radius of the coils, $x$ is the distance between the measurement point to the center of the coil, $N$ is the number of turns of the coils, and $I$ is the current intensity through the coils. Therefore, the magnetic field intensity $B_0$ at the coil center is:

$$ B_0 = \frac{\mu_0}{2\bar{R}} N \cdot I \quad (2) $$

The formula for calculating off-axis magnetic field distribution is complicated and is omitted.

Helmholtz coils are a pair of parallel, coaxial, and circular coils. The current in the two coils is in the same direction with equal intensity. The distance, $d$, between the two coils is equal to the radius, $R$, of the circular coils. Helmholtz coils are known for creating a uniform magnetic field in the region around the middle of their common axis, so they are commonly used to measure weak magnetic fields.

In general, the magnetic field intensity at any point on the axis of the Helmholtz coils is:

$$ B' = \frac{1}{2} \mu_0 \cdot N \cdot I \cdot R^2 \left\{ \left[ R^2 + \left( \frac{R}{2} + z \right)^2 \right]^{-3/2} + \left[ R^2 + \left( \frac{R}{2} - z \right)^2 \right]^{-3/2} \right\} \quad (3) $$

where $z$ is the displacement of the measurement point from the center of the Helmholtz coils. At the center of the Helmholtz coils, the magnetic field intensity is:

$$ B'_0 = \frac{8}{5^{3/2}} \frac{\mu_0 \cdot N \cdot I}{R} \quad (4) $$