2. Theory

When an electric current passes through a conductor along the direction perpendicular to an external magnetic field, an electric potential difference is created in the direction perpendicular to both the current and the magnetic field across the two ends of the conductor. This phenomenon is known as the Hall effect, found by American physicist Hall in 1879. In general, the Hall effect in metals and electrolytes is very small, but is more notable in semiconductors such as N-type germanium, InSb, phosphorus indium arsenide, or gallium arsenide (GaAs). GaAs Hall elements are usually used for the measurement of magnetic fields due to their high sensitivity, wide linear range, and lower temperature coefficient. Hall elements are known for ease of use, small probe size, and suitability for magnetic field measurement in a small region. The relative uncertainty of the Hall-effect technique is about 10^{-2}-10^{-3}. This apparatus helps students deepen their understanding on the basic principle of Hall effect; learn how to measure the sensitivity of a Hall element and magnetic field using a Hall element.

A. Hall Effect

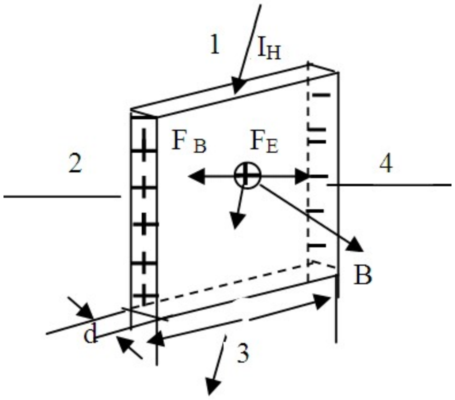

Figure 1 Schematic diagram of the generation of Hall voltage.

As shown in Fig. 1, when an electric current I_H goes through a p-type Hall element for example, holes have a certain drift velocity υ, and thus the applied perpendicular magnetic field *B* creates a Lorentz force *F* on the moving charge (carrier):

$$\vec{F}_B = q\ (\vec{\upsilon} \times \vec{B}) \tag{1}$$

where *q* is the electron charge. The Lorentz force deflects the carriers in horizontal direction to accumulate at the border of the sample creating a transverse electric field *E*. This accumulation process continues until the electric field ($F_E = qE$) and the magnetic field (F_B) forces exerted on the carrier are canceled out, as

$$q\ (\vec{\upsilon} \times \vec{B}) = q\vec{E} \tag{2}$$

As a result, the charges are no longer deflected in the sample, creating a Hall electric potential difference by the electric field.

For an n-type sample, the transverse electric field is opposite to that of a p-type sample, so that the Hall potential differences of n-type and p-type samples have opposite signs that can be used to determine the conductive type of a Hall element.

If a p-type sample has carrier concentration p, width ω and thickness d; and the current through the sample is I_H ($I_H = pq\upsilon\omega d$), then the speed of holes is $\upsilon = I_H/pq\omega d$. From Eq. (2), we have:

$$E = \left| \vec{\upsilon} \times \vec{B} \right| = I_H B / pq\omega d \tag{3}$$

If Eq. (3) is multiplied by ω on both sides, we get:

$$U_H = E\omega = I_H B/pqd = R_H \times I_H B/d \tag{4}$$

where $R_H = 1/pq$ is called the Hall coefficient. In practice, Eq. (4) is usually written as:

$$U_H = I_H K_H B \tag{5}$$

where coefficient $K_H = R_H/d = 1/pqd$ is the sensitivity of a Hall element in unit of mV/(mA·T). Generally, the larger K_H is the better. As K_H is inversely proportional to carrier concentration p and the carrier concentration in semiconductor is far smaller than that in metal, semiconductor materials are preferred for making Hall elements. K_H is also inversely proportional to thickness d, so Hall elements are made very thin, only 0.2 mm in thickness.

From Eq. (5), once Hall sensitivity K_H is known, both Hall current I_H and Hall voltage U_H are measured, magnetic field strength B can be hence calculated. This is the basis of magnetic field measurement using a Hall element.

B. Magnetic Field Measurement using Hall Element

There are a few methods for the measurement of magnetic field strength, such as magnetic flux method, NMR method and Hall effect method. Among them, Hall effect method can measure AC and DC magnetic fields, and is the simplest, most intuitive and fastest method.

As shown in Fig. 2, a DC electric source E_1 provides adjustable magnetizing current I_M to an electromagnet through a variable resistor R_1. An electric source E_2 provides Hall current I_H to a Hall element through a variable resistor R_2. Source E_2 can be either DC or AC. A voltmeter is used to measure the Hall current I_H and Hall voltage U_H.

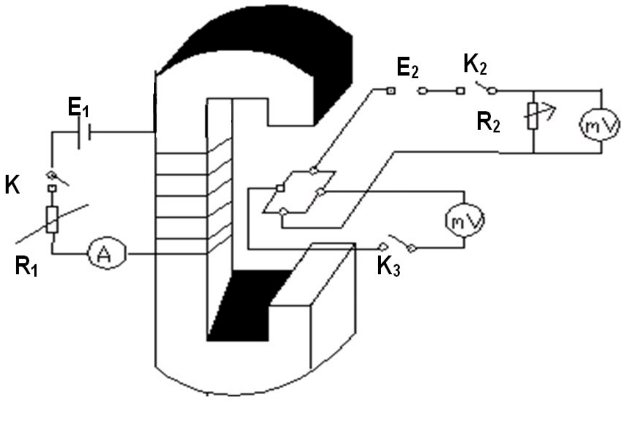

Figure 2 Circuit schematic of measuring Hall potential difference

Semiconductor materials can be categorized into two types, n-type (electron-type) and p-type (hole-type). The carriers of the former are electrons with negative charges while the carriers of the later are holes equivalent to positively charged particles. From Figure 1, it can be seen that if the carriers are electrons, the potential at point 4 is higher than that at point 3, i.e. $U_{H3\cdot 4} < 0$; if the carriers are holes, the potential at point 4 is lower than that at point 3, i.e. $U_{H3\cdot 4} > 0$. If the carrier type of the material is known, the magnetic field direction can be determined based on the sign of potential U_H.

As the time required for the creation of an electric field via Hall effect is very short (10^{-12}~10^{-14} s), both AC and DC current can be used by a Hall element. If Hall current is $I_H = I_0\sin(\omega t)$, then we have:

$$U_H = I_H K_H B = I_0 K_H B\sin(\omega t) \tag{6}$$

The resulted Hall potential is also alternating. In AC case, Eq. (5) is still valid but I_H and U_H should be interpreted as effective values.

C. Side Effect Elimination of Hall Element

In reality, there are thermal magnetic side effects that contribute to unwanted voltage added to Hall voltage U_H leading to measurement errors.

These include: (a) Ettingshausen effect for the creation of a thermoelectric potential U_E due to the existence of a temperature difference at the two sides of a Hall element. It is related to the directions of Hall current I_H and magnetic field B; (b) Nernst effect for the creation of a potential U_N between the two sides (3&4) of a Hall element when heat flows through the Hall element (e.g. terminals 1&2). It is only related to magnetic field B and heat flux; and (c) Righi–Leduc effect for the creation of a thermoelectric potential U_R due to a temperature difference at the two ends of a Hall element created by heat flowing through the Hall element. It is related to magnetic field B and the heat field.

Apart from these thermal magnetic side effects, there exists an unequal potential difference U_0. It is caused by two electrodes at the two sides (3&4) of a Hall element not locating at the same equipotential surface. Under such case, when a Hall current passes through terminals 1 & 2 of a Hall element even in the absence of a magnetic field, a potential difference U_0 is still created between terminals 3 & 4. Its direction changes with the direction of current I_H.

To eliminate the impacts of these side effects, the directions of I_H and B need to be changed in the experiment and 4 sets of potential values should be recorded (positive if toggle switch K_1or K_2 is set at upright position) as

$U_1 = U_H + U_0 + U_E + U_N + U_R$, when both I_H and B are positive

$U_2 = -U_H - U_0 - U_E + U_N + U_R$, when I_H is negative but B is positive

$U_3 = U_H - U_0 + U_E - U_N - U_R$, when both I_H and B are negative

$U_4 = -U_H + U_0 - U_E - U_N - U_R$, when I_H is positive but B is negative.

By calculating $U_1 - U_2 + U_3 - U_4$ and taking average, we have:

$$(U_1 - U_2 + U_3 - U_4)/4 = U_H + U_E \tag{7}$$

Since $U_E \ll U_H$, Eq. (7) becomes:

$$U_H = (U_1 - U_2 + U_3 - U_4)/4. \tag{8}$$

Alternatively, Eq. (8) can be written as:

$$U_H=(|U_1|+|U_2|+|U_3|+|U_4|)/4 \tag{9}$$

where the sign of U_H is determined by the type of the Hall element, p-type or n-type.