2. Theory

In this apparatus, there are two electrodes placed in a vacuum glass tube, namely as cathode and anode. The cathode is made of a metal wire that is heated by an electric current; when a positive potential is applied from anode to cathode, an electric current will flow through the external circuit connecting the two electrodes. Electron emission due to heating is normally called thermionic emission, while work function or electron escape potential is a fundamental physical quantity of thermionic emission governed by the Richardson's law.

A. Determination of electron work function

In quantum physics, free electrons in metal follow the following laws: (1) energy levels of free electrons in metal are quantized; (2) all electrons are indistinguishable; and (3) their filling levels follow the Pauli exclusion principle. Furthermore, the energy distribution of electrons in metal obeys the Fermi-Dirac distribution. At absolute zero temperature, the electron energy distribution shown in Figure 1, has a maximum kinetic energy of *W*i, known as the Fermi level; when temperature rises, the electron energy distribution changes as shown in Figure 1 (some electrons possess a kinetic energy higher than *W*i, and the number of free electrons decreases exponentially with energy).

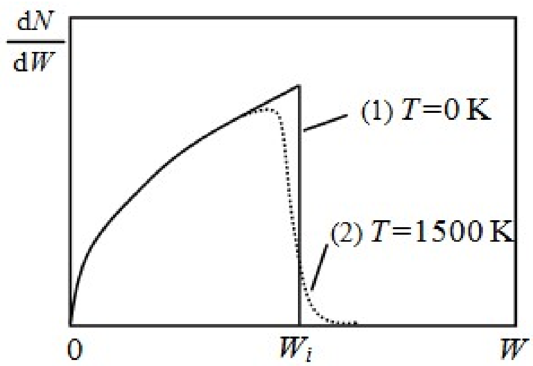

Figure 1 Electron energy distribution

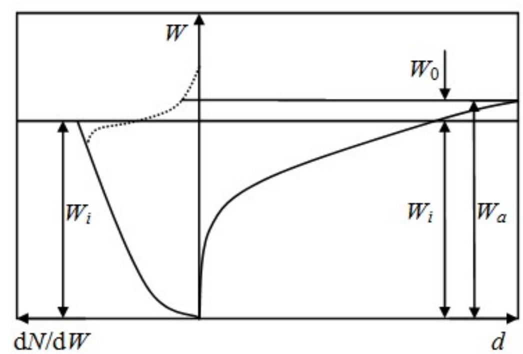

Figure 2 Potential barrier of metal

Due to the existence of an electron-positive charge dipole layer of about 10^{-10} m in thickness on metal surface, there is a potential barrier *W*a on metal surface for electron to overcome in order to escape from the metal. The minimum energy required by electron to escape from a metal surface is called the work function of the metal, represented by W_0, where $W_0=W_a-W_i=e_0\varphi$, in unit of eV. φ is called the escape potential.

The current of thermionic emission, i.e. Richardson-Dushman formula, can be derived as:

$$I_e = AST^2 \exp^{-\frac{e\varphi}{kT}} \tag{1}$$

where *S* is the effective area of cathode in unit square centimeter; *T* is the absolute temperature of cathode in degrees Kelvin; $e\varphi$ is the work function of cathode metal in unit electron-volt; *k* is the Boltzmann constant; and *A* is a coefficient related to the chemical purity of the cathode.

a) Parameters *A* and *S*

Parameter A is directly related to the electron reflection coefficient R_e of the metal surface, which is strongly related to the chemical purity of the metal surface, and hence the potential energy barrier. If the metal surface is not clean enough or the vacuum level of the electric tube is not high enough, R_e can vary directly affecting parameter A. Also, if the metal surface is rough, the calculated emission area and the actual area S may also differ.

To eliminate the above uncertainty caused by parameters A and S, we can divide equation (1) by T^2 at both sides, and take logarithmic transformation as:

$$\lg \frac{I_e}{T^2} = \lg AS - 5.039 \times 10^3 \frac{\varphi}{T} \tag{2}$$

From (2), it can be seen, $\lg \frac{I_e}{T^2}$ and $\frac{1}{T}$ are in a linear relationship. By plotting $\lg \frac{I_e}{T^2} \sim \frac{1}{T}$ graph and doing linear curve-fitting to the data, φ can be determined from the slope of the fitted line. Now that term *lgAS* only changes the intercept of the fitted line and does not affect the slope of the fitted line, the uncertainty of A and S is eliminated.

b) Measurement of emission current I_e

When electrons emitted from the cathode reach the anode, they form an electric field between anode and cathode, which adversely affects further electron migration from cathode to anode. To overcome this issue, a positive electric potential must be maintained from anode to cathode by introducing an acceleration field E_a between anode and cathode, as shown in Figure 3

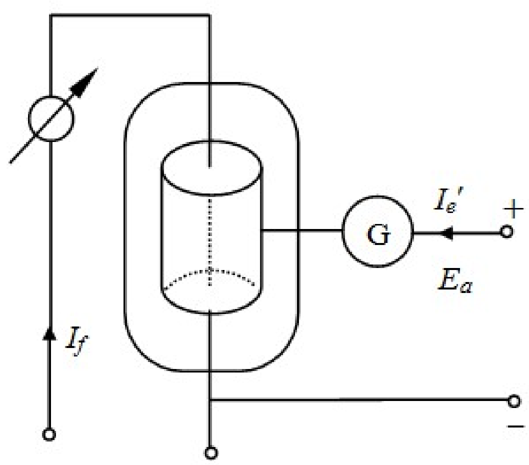

Figure 3 Schematic of acceleration field between anode and cathode

On the other hand, the external acceleration field affects the emission of thermal electrons by increasing the emission current through the so-called Schottky effect as

$$I_e' = I_e e^{\frac{0.439\sqrt{E_a}}{T}} \tag{3}$$

where $I_{e'}$ and I_e are the emission current in the presence and absence of an acceleration field, respectively. By taking logarithmic operation to (3), we get:

$$\lg I'_e = \lg I_e + \frac{0.439}{2.303T}\sqrt{E_a} \tag{4}$$

If both cathode and anode are co-axial cylinders with radii of r_1 and r_2, respectively; Ua is anode voltage, the acceleration filed can be written as:

$$E_a = \frac{U_a}{r_1(\ln r_2 - \ln r_1)} \tag{5}$$

Equation (4) becomes:

$$\lg I'_e = \lg I_e + \frac{0.439}{2.303T}\frac{1}{\sqrt{r_1(\ln r_2 - \ln r_1)}}\sqrt{U_a} \tag{6}$$

It is apparent from (6) that $\lg I'_e$ and $\sqrt{U_a}$ are in a linear relationship. By plotting $\lg I'_e \sim \sqrt{U_a}$ graph and doing linear curve-fitting to the data, the intercept of the fitted line is lgI_e, which varies with temperature.

c) Measurement of temperature *T*

In this experiment, cathode temperature is determined from cathode heating current (filament current), as shown in Table 1.

Table 1 Relationship between filament current and filament temperature

Current (A)	0.650	0.675	0.700	0.725	0.750	0.775	0.800
Temperature (10^3K)	1.96	2.00	2.04	2.08	2.12	2.16	2.20

The electric tube used in this apparatus is an ideal diode as shown in Figure 4. The cathode is a tungsten wire while the anode is cylindrical electrode (radius 4.0 mm) made from nickel plate. There is a small hole on the anode with the purpose of measuring filament temperature using an optical pyrometer. To avoid the cold side effect of the filament and the edge effect of the electric field, two protective electrodes are introduced at both ends of the anode. Protective electrodes are applied with the same voltage as the anode, but their current is not counted in thermionic emission current.

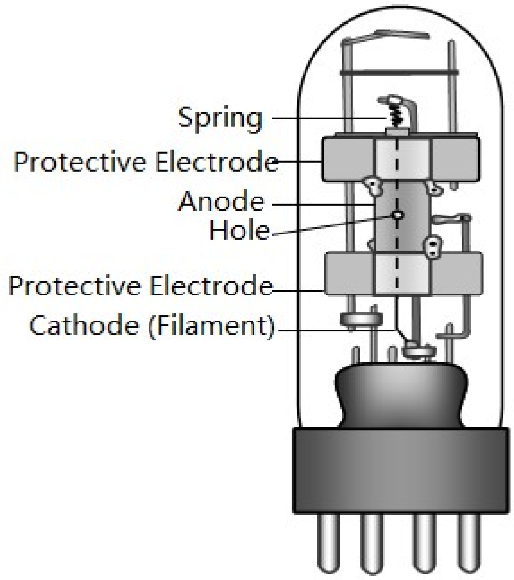

Figure 4 Schematic of ideal diode

B. Measurement of specific charge of electron using magnetron method

In an ideal diode, both cathode and anode are coaxial cylinders. When the anode has a positive voltage, electrons emitted from the cathode undertake a radial motion due to the impact of the electric field, as shown in Figure 5 (a). If a magnetization coil is placed outside of the ideal diode, the radial motion of electrons is bent by the impact of the axial magnetic field, as shown in Figure 5 (b). By further increasing magnetic field (i.e. increasing the magnetization current), electron trajectory is bent more significantly, as shown in Figure 5 (c). If the Lorenz force minus the electric field force exerted on electrons equals the centripetal force of electrons near anode, electrons undertake circular motion along the inner wall of the anode. Under such case, electrons reach a state called "critical state". By further enhancing the magnetic field, radius of electron motion will reduce, so that electrons can no longer arrive at the anode and hence no current is created, as shown in Figure 5 (d). In general, the method of using magnetic field to control anode current is called "magnetron".

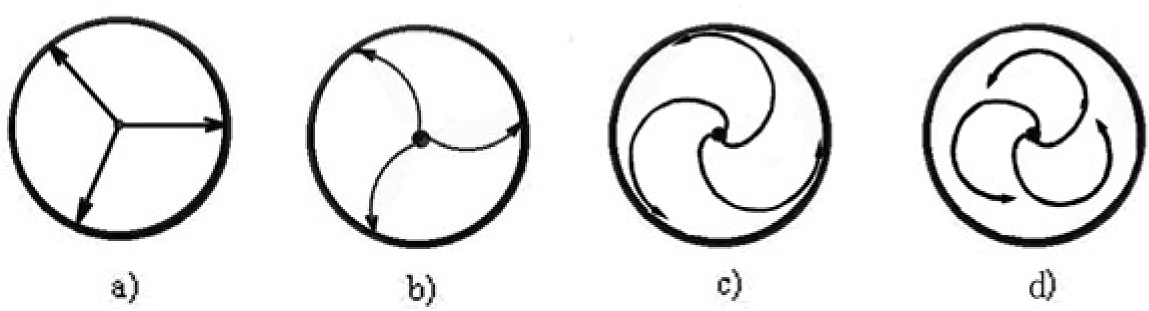

Figure 5 Motion trajectory of electrons in ideal diode under different magnetic field strengths

At a certain acceleration voltage, the relationship between anode current I_a and magnetization current I_s is shown in Figure 6. As shown in Figure 6, in segment 1-2, the anode current remains unchanged corresponding to Figures 5 (a) and 5 (b); in segment 2-3, the rate of change in anode current reaches maximum corresponding to Figure 5 (c); after segment 2-3, anode current gradually decreases with an increase in magnetization current until zero. In Figure 6, point Q, whose height is $I_{a0}/4$, is called the "critical point".

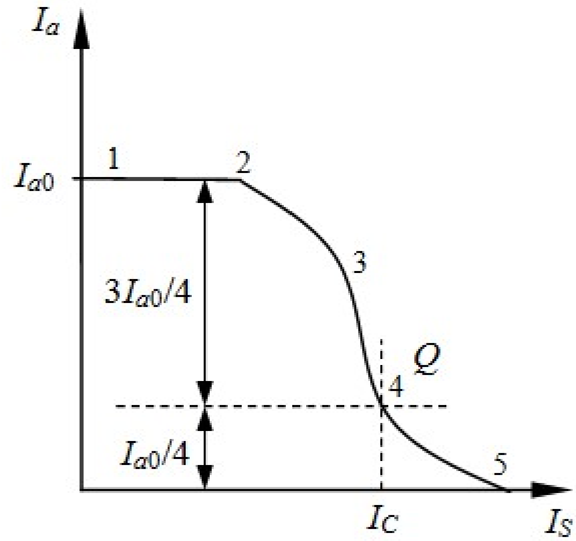

Figure 6 Relationship between anode current and magnetization current

In an approximation of a single electron, the kinetic energy of an electron emitted from the cathode is the combination of anode acceleration energy eU_a and "thermal motion" energy W from heated filament, as

$$\frac{1}{2}mv^2 = eU_a + W \tag{7}$$

where m, e, and ν are the mass, charge, and velocity of the electron, respectively.

In a magnetic field, the electron will take a circular motion with radius R,

$$m\frac{v^2}{R} = evB \tag{8}$$

where B is the strength of the magnetic field.

The magnetic field strength in the center of a magnetization coil with current I_s is:

$$B = \frac{\mu_0 N I_S}{2(r_2 - r_1)} \ln \frac{r_2 + \sqrt{r_2^2 + L^2}}{r_1 + \sqrt{r_1^2 + L^2}} = K' I_S \tag{9}$$

From (7), (8) and (9), we have:

$$\frac{U_a + W/e}{I_S^2} = \frac{e}{m} \cdot \frac{R^2}{2} \cdot K'^2 \tag{10}$$

When most electrons are in critical state while ignoring the radius of cathode (filament), the magnetization current corresponding point Q is called critical current I_c ($R=a/2$, where a is the inner radius of the anode). The relationship between anode voltage U_a and I_c can be written as:

$$\frac{U_a + W/e}{I_C^{\ 2}} = \frac{e}{m} \cdot \frac{a^2}{8} \cdot K'^2 = K \qquad (11)$$

Apparently, U_a and I_C^2 are in a linear relationship.

For an ideal diode, the relationship curve of anode current vs magnetization current varies with anode voltage (U_a) as seen in Figure 7, so critical current (I_c) also varies with anode voltage (U_a). By plotting $U_a \sim I_c^2$ and doing linear curve-fitting to the data, slope K can be acquired from the fitted line. Hence, the motion laws of electron in radial electric field and axial magnetic field can be verified.

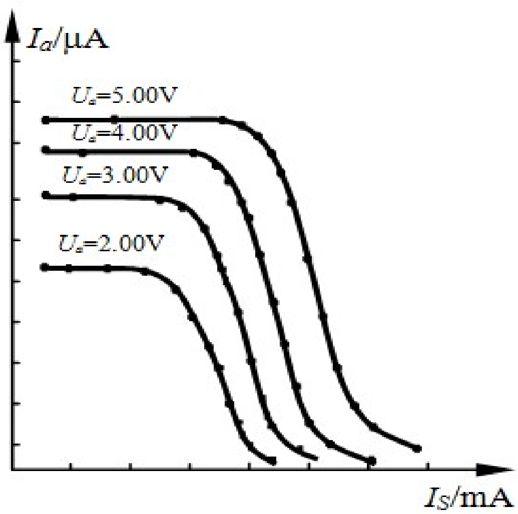

Figure 7 Relationship curve of anode current vs magnetization current

The parameters of magnetization coil used in this apparatus are as follows: inner radius r_1=24.0 mm, outer radius r_2=36.0 mm, length L=18.0 mm, number of turns N=800 while permeability in vacuum is $\mu_0=4\pi\times10^{-7}$ H/m. So the magnetic field strength at the center of coil is:

$$B = \frac{\mu_0 N I_C}{2(r_2 - r_1)} \ln \frac{r_2 + \sqrt{r_2^2 + L^2}}{r_1 + \sqrt{r_1^2 + L^2}} = 1.445 \times 10^{-2} I_C$$

Thus, we have $K'=1.445\times10^{-2}$ for (11), and the charge-mass ratio (e/m) of electron can be obtained.