## 2. Theory

LEEI-15, Apparatus of Measuring Liquid Conductivity, measures the conductivity of liquid using a hollow mutual inductance sensor. The interior of the sensor is composed of two inductive coils of nano-material iron-based alloy rings. Each ring is wound with a group of coil, and the number of turns of the two coils is the same. The schematic diagram is shown in Figure 1.

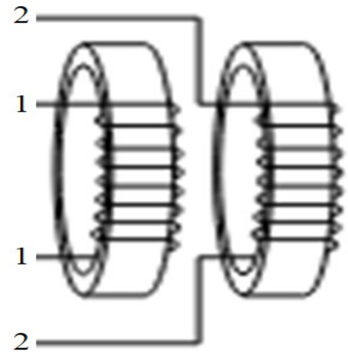


Figure 1 Structure diagram of conductivity measurement sensor

The working principle of this sensor is: the alternating current output by the signal generator generates an alternating magnetic field in ring of the coil (1, 1), which generates an alternating induced current in the conductive liquid. The induced current generates an alternating magnetic field in the ring of the coil (2, 2) which is also immersed in the liquid. This magnetic field generates an induced electromotive force in the coil (2, 2), which becomes the output signal of the sensor.

Changing the conductivity (σ) of the liquid, under the same input amplitude ($V_{in}$), the induced current will change, resulting in a change in the output signal voltage ($V_{out}$) of the sensor. It can be proved that the conductivity σ of the liquid is proportional to $V_{out}$ within a certain range of $V_{in}$, so it can be written as:

$$\sigma = K(V_{out}/V_{in}) . \tag{1}$$

$(V_{out}/V_{in})$ is called "voltage decay". So under a certain input amplitude, the conductivity is proportional to the voltage decay.

In the measuring apparatus, the container holding the liquid to be measured is very large. The amplitude of $V_{out}$ is mainly related to the liquid in the hollow cylinder of the sensor (abbreviated as liquid column). The conductivity of the liquid can be calculated from the liquid column. The resistance of the liquid column of the sensor is equivalent to a solid resistance $R = \rho \frac{L}{S} = \frac{1}{\sigma}\frac{L}{S}$, so:

$$\sigma = \frac{1}{R}\frac{L}{S} . \tag{2}$$

Comparing (1) and (2), we get:

$$V_{out}/V_{in} = \frac{1}{K}\frac{L}{S}\frac{1}{R} = B\frac{1}{R} . \tag{3}$$

In formula (3), $B = \frac{1}{K}\frac{L}{S}$, can also be written as $K = \frac{1}{B}\frac{L}{S}$. Substitute it into (1), we get:

$$\sigma = (\frac{1}{B}\frac{L}{S})V_{out}/V_{in} \quad . \tag{4}$$

This demonstrates that the liquid conductivity is related to the length (*L*), cross-sectional area (*S*), voltage decay $(V_{out}/V_{in})$ and proportional constant (*B*) of the hollow cylinder. Since outside of the hollow cylinder, the equivalent *S* of liquid is large, *R* is very small. Therefore, the induced current in the liquid is mainly limited by the resistance of the liquid in the hollow cylinder.

It should be noted that in the experiment, in order to calibrate the proportional constant (*B*) at multiple points, a variety of standard σ liquids need to be used, which makes the operation time-consuming and difficult. Therefore, according to the above principle, using a resistance loop, also called "check standard" (*R*), to replace the standard σ liquid to do the calibration in experiment is more convenient and accurate.

In the process of doing calibration with the "check standard" resistance, we use a conductive wire to short the two ends of the standard resistance to make a resistance loop. It should be noted that part of the resistance loop must pass through the hollow cylinder of the sensor.

Experiment proves that the error between using the "check standard" and using the standard brine is less than $10^{-3}$. In addition, the conductivity of the standard liquid is sensitive to temperature, so in practice, standard brine is not used for calibration.