3. Theory

A. Magnetization law of ferromagnetic materials

The phenomenon of generating a new magnetic field by applying an external magnetic field to a material is called magnetism. The magnetism of materials can be classified into three types, anti-magnetism (diamagnetism), para-magnetism, and ferromagnetism. Materials that can be magnetized are called magnetic media. In ferromagnetic material, there exists a strong inter-coupling effect among adjacent electrons. In the absence of an external magnetic field, their spin magnetic moments can be spontaneously arranged in order within a tiny region to form spontaneously magnetized small regions, known as magnetic domains. In an un-magnetized material, although each magnetic domain has magnetism, the whole material does not show net magnetism as each magnetic domain has random magnetization direction and the vast average is zero. A structural diagram of poly-crystalline magnetic domains is shown in Figure 2 (a).

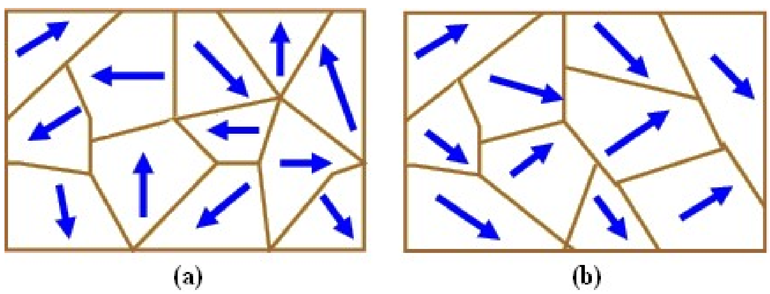

Figure 2 Poly-crystalline magnetic domains (a) before and (b) after magnetization

When the ferromagnetic material is placed in an external magnetic field, these small magnetic domains tend to rotate to follow the direction of the external field by varying their volumes. At this moment, the ferromagnetic material shows macro magnetism. This effect increases with an increase in the intensity of the external magnetic field, until all magnetic domains are arranged along the external magnetic field. At this moment, the magnetization of the material reaches saturation. Figure 2 (b) displays the arrangement of polycrystalline magnetic domains after magnetization.

Because the magnetic moment in each domain is arranged in order, the material shows strong magnetism. This is the reason why the magnetism of ferromagnetic materials is much stronger than that of paramagnetic materials. Doping and internal stress inside the material prevent these magnetic domains from recovering to their original states after the external magnetic field is withdrawn. This is the main cause for hysteresis phenomenon. Ferromagnetism is inseparable with magnetic domain structures. If a ferromagnetic material is impacted by strong vibrations or heated to a high temperature, its magnetic domains collapse. At this moment, a series of ferromagnetic properties (such as high magnetic permeability and hysteresis) which are related to magnetic domains disappear. All ferromagnetic materials have such a critical temperature. When the applied temperature is higher than this critical temperature, their ferromagnetism disappears and the materials become para-magnetism. This critical temperature is called Curie point of ferromagnetic materials.

In various magnetic media, the most important material is represented by iron kind of strong-magnetic substances. Except for iron, general transition metals (such as cobalt and nickel) and lanthanides such as dysprosium and holmium have ferromagnetism. However, most commonly used ferromagnetic materials are alloys of iron and other metallic or non-metallic components, and some iron oxides (ferrites). Ferrites have characteristics of high resistivity and low Eddy current losses, and are suitable for use at high frequency. The main constituent of one kind of soft ferrites is Fe_2O_3. Its general formula can be expressed as $MO\cdot Fe_2O_3$ (spinel-type ferrite), where M is the 2^+ metal element whose spontaneous magnetization is sub-ferrimagnetism. Currently, magnet core materials are mainly ferrites with Ni-Zn centers.

Magnetization law of magnetic media is described by magnetic induction *B*, magnetization *M* and magnetic field strength *H*. They satisfy the following relations:

$$B = \mu_0(H + M) = (\chi_m + 1)\mu_0 H = \mu_r \mu_0 H = \mu H \quad (1)$$

where $\mu_0 = 4\pi\times10^{-7}$ *H*/m is the vacuum permeability, x_m is the magnetic susceptibility, μ_r is the relative permeability (a dimensionless coefficient), and μ is the absolute permeability. For paramagnetic media, $x_m>0$, μ is slightly larger than 1; for diamagnetic media, $x_m<0$ and in general their absolute values are between $10^{-4} \sim 10^{-5}$, μ is slightly less than 1; for ferromagnetic media, $x_m>>1$, so $\mu_r>>1$.

For isotropic non-ferromagnetic media, there exists a linear relationship between *H* and *B*, i.e. $B=\mu H$; while for ferromagnetic media, there has a complex non-linear relationship among μ, *B* and *H*. Under normal conditions, spontaneous magnetization exists in ferromagnetic materials, and the lower the temperature is, the greater the spontaneous magnetization will be.

A typical magnetization curve (*B-H* curve) is shown in Figure 3, which reflects the common characteristics of magnetic susceptibility of ferromagnetic materials, i.e. at the beginning, *B* increases slowly as *H* increases when μ is relatively small; then *B* increases sharply as *H* increases and μ also increases rapidly; finally *B* tends to saturate as *H* increases, at this time, μ reaches the maximum value and then decreases quickly. Figure 4 indicates that magnetic permeability μ is a function of magnetic field *H*.

Permeability is also a function of temperature as shown in Figure 4. When temperature reaches a certain value, a ferromagnetic material changes its magnetic state from ferromagnetism to paramagnetism. The mutation point on the temperature curve is the Curie temperature T_c.

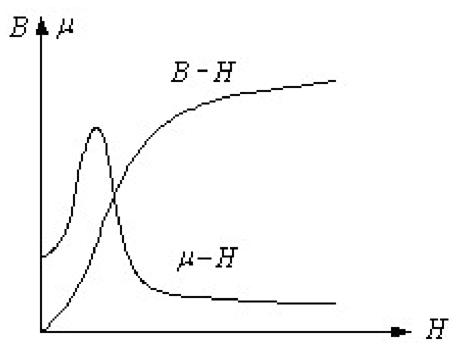

Figure 3 Magnetization curve and $\mu\sim H$ curve

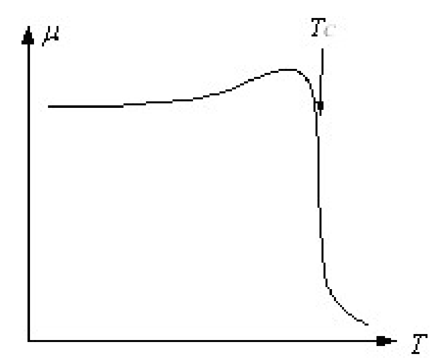

Figure 4 $\mu\sim T$ curve

B. Hysteresis phenomenon of ferromagnetic material

The magnetization process of ferromagnetic materials is so complex that normally it is studied by measuring the relationship of magnetic field strength *H* versus magnetic induction intensity *B* of the magnetization field.

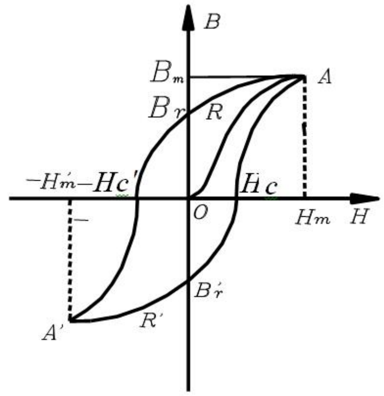

Figure 5 Hysteresis loop and magnetization curve

As shown in Figure 5, if there is no magnetization field in a ferromagnetic material, both *H* and *B* are zero, corresponding to the coordinate origin *O* in the *B*-*H* graph. As the magnetic field strength *H*, increases, the magnetic induction intensity *B* also increases, but not linearly with *H*. When *H* increases to a certain value, *B* no longer increases or increases very slowly indicating that the magnetization of the material has reached saturation. H_m and B_m are the saturated magnetic field strength and induction intensity, respectively, as shown by point *A* in Figure 5.

If *H* is gradually reduced to zero, *B* decreases accordingly. However, the trajectory of the *H*-*B* curve does not follow the original curve (*AO*) to return to point *O*; instead it follows another curve *AR* to reach to point B_r suggesting residual magnetism still remained in the ferromagnetic material even when *H* drops to zero. However, if magnetic field *H* is reversed and gradually increased until H=-H_m, the curve reaches point A' (reversal saturation point). Now reduce the magnetization field back to *H*=0 and then change magnetic field *H* to positive while gradually increasing to saturation (H_m). Curve *A'R'A* that is symmetric with *ARA*' should be achieved. The closed loop from point *A* via *ARA*', *A'R'A* back to point *A* is called the magnetic hysteresis loop of a ferromagnetic substance (saturated magnetic hysteresis). The intersection points of the curve with *H* axis, H_0 and H'_0, are called the coercivity, while the intersection points on *B* axis, B_r and B'_r, are known as the residual magnetic induction intensity.

B. Using oscilloscope to observe the dynamic hysteresis loop

The circuit schematic is shown in Figure 6. The ferromagnetic sample is made into a closed loop. The magnetizing coil N_1 and secondary coil N_2 are evenly wound on it. An AC voltage u is applied to the magnetizing coil, and a sampling resistor R_1 is connected in series in the circuit. The voltage u_1 on R_1 is connected to the X-axis input terminal CH1 of the oscilloscope. The secondary coil N_2 is connected in series with the resistor R_2 and capacitor C in series. The voltage u_2 across the capacitor C is applied to the Y-axis input CH2 of the oscilloscope. Such a circuit can display and measure the hysteresis loop of the ferromagnetic material on the oscilloscope.

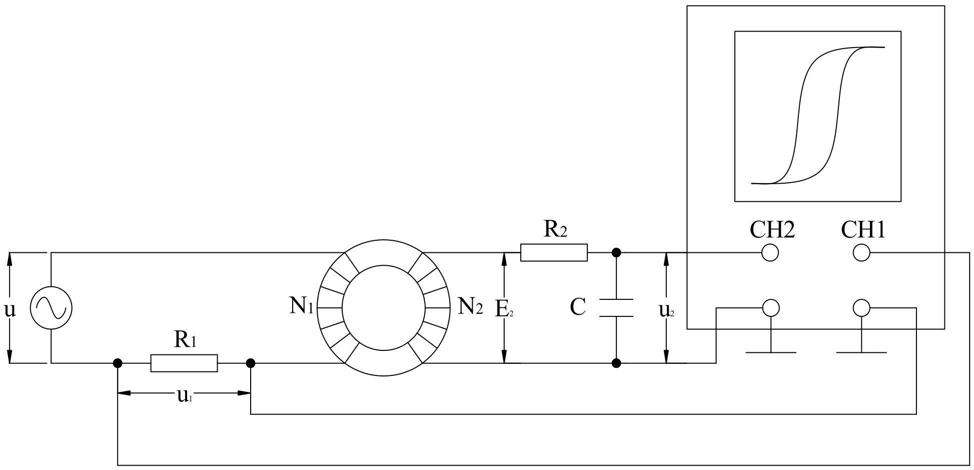

Figure 6 Circuit diagram for measuring dynamic hysteresis loop with an oscilloscope

C. Measurement of magnetic field strength H

Suppose the average circumference of the ring sample is l, the number of turns of the magnetizing coil is N_1, and the magnetizing current i_1 is AC sine wave. According to the Ampere's circuital law, we have $Hl = N_1 i_1$. With $u_1 = R_1 i_1$, we get:

$$H = \frac{N_1 \cdot u_1}{l \cdot R_1}, \tag{2}$$

where u_1 is the voltage on the sampling resistor R_1.

From formula (2), under the conditions of R_1, l, and N_1 are known, the value of the magnetic field strength H can be calculated by formula (2) when u_1 is measured..

D. Measurement of magnetic induction strength B

Suppose the cross-sectional area of the sample is S, according to the law of electromagnetic induction, the induced electromotive force in the secondary coil with the number of turns N_2 is

$$E_2 = -N_2 S \frac{dB}{dt}, \tag{3}$$

where $\frac{dB}{dt}$ is the derivative of magnetic induction intensity B with time.

If the current in the circuit connected to the secondary coil is i_2 and the quantity of electricity on the capacitor C is Q, then there is

$$E_2 = R_2 i_2 + \frac{Q}{C} \ . \tag{4}$$

In equation (4), considering that the secondary coil turns are not too many, the self-induced electromotive force is negligible. When selecting circuit parameters, both R_2 and C are taken to larger values to make the voltage drop across the capacitor $u_2 = \frac{Q}{C} << R_2 i_2$ negligible, so equation (4) can be written as

$$E_2 = R_2 i_2 \ . \tag{5}$$

Substitute the current $i_2 = \frac{dQ}{dt} = C\frac{du_2}{dt}$ into (5), we get

$$E_2 = R_2 C \frac{du_2}{dt} \ . \tag{6}$$

Substituting (6) into (3), we have $-N_2 S\frac{dB}{dt} = R_2 C\frac{du_2}{dt}$. When integrating both sides of this formula over time, since both B and u_2 are alternating, the integral constant term is zero. Therefore, without considering the minus sign (here only referring to the phase difference $\pm\pi$), the magnetic induction strength is:

$$B = \frac{R_2 C u_2}{N_2 S} \ , \tag{7}$$

In formula (7), N_2, S, R_2 and C are all constants. By measuring the amplitude of the voltage u_2 across the capacitor and substituting it into formula (7), the value of the magnetic induction strength B of the material can be obtained.

If the frequency is not too slow, when the magnetizing current changes for one period, the light spot on the oscilloscope will depict a complete hysteresis loop, and this process will be repeated every subsequent period to form a stable hysteresis loop.