2. Theory

A schematic of the electron diffraction tube of the apparatus is shown in Figure 1. A circular crystalline metal film target with a diameter of 15 mm is placed between the electron gun and the screen in the diffraction tube. Electron diffraction pattern is formed by the direct hitting of an electron beam on the crystal surface.

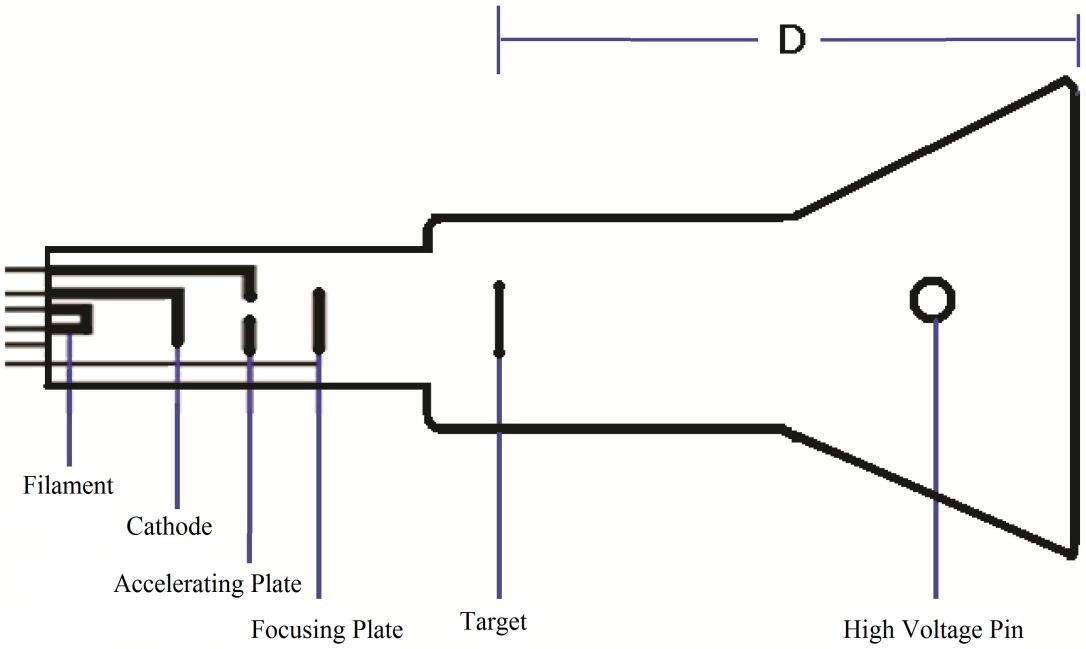

Figure 1 Schematic of electron diffraction tube

In the electron gun portion, electron beam is emitted from the cathode which is heated by the filament, is further accelerated by an electric field of approximate 20 kV. After electrostatic focusing and deflection, the electron beam is focused onto the target surface.

If an electron beam passing through the crystal film at speed v, the de Broglie wavelength of the electron beam is:

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{1}$$

where h is Planck's constant, $p = mv$ is the momentum of the electron, and m is the mass of the electron. Since the kinetic energy of the electron is:

$$\frac{1}{2}mv^2 = eV \tag{2}$$

where e is the charge of the electron and V is the accelerating voltage, the de Broglie wavelength of the electron beam can be rewritten as:

$$\lambda = \frac{h}{\sqrt{2meV}} \tag{3}$$

Since m=9.109×10^{-31} kg, e=1.602×10^{-19} C, and h=6.626×10^{-34} m^2 kg/s, we get:

$$\lambda \approx \left(\frac{1.50}{V}\right)^{1/2} \tag{4}$$

where λ is in unit of nanometer (nm), and V is in unit of volt (V). Since atoms are aligned regularly in a crystal, they form a series of parallel planes in various directions with each serial parallel plane presented by Miller indices (h, k, l). By considering an electron beam hitting on a serial parallel plane of the atom structure, as shown in Figure 2, the condition of the beam exiting from the film is:

$$n\lambda = 2d\sin\theta \tag{5}$$

where n is an integer, θ is the angle between the diffracted direction and the incident direction of the electron beam, and d is the distance between two adjacent parallel planes.

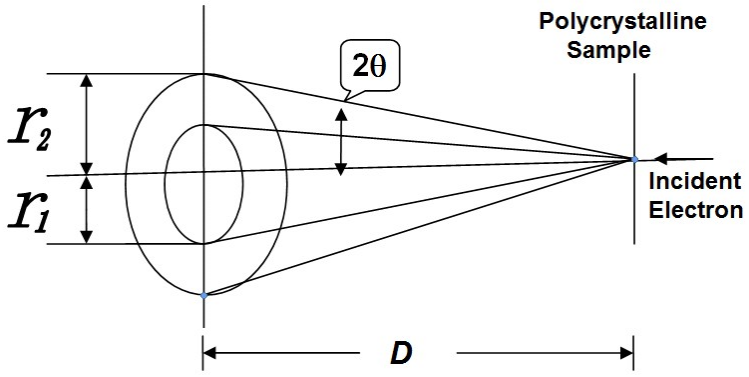

Figure 2 Schematic of electron beam diffraction from a crystal film

If θ is small, $\sin\theta$ can be approximated by θ=r/2D (small angle approximation), where r is the radius of a diffraction ring, and D is the distance between the target and the screen.

The distance between two parallel planes of Miller indices (*h, k, l*) is:

$$d = \frac{a}{(h^2 + k^2 + l^2)^{\frac{1}{2}}} \tag{6}$$

where a is the lattice constant, i.e. the edge length of a single crystal cell. By substituting (6) into (5), we get

$$\lambda = \frac{2a \sin\theta}{n(h^2 + k^2 + l^2)^{\frac{1}{2}}} \tag{7}$$

Let $H = nh$, $K = nk$ and $L = nl$, we have

$$\lambda = \frac{2a \sin\theta}{(H^2 + K^2 + L^2)^{\frac{1}{2}}} = \frac{r}{D} \bullet \frac{a}{(H^2 + K^2 + L^2)^{\frac{1}{2}}} \tag{8}$$

This means that the n^{th} order Bragg diffraction of any plane with Miller indices (*h, k, l*) can be considered as the first order Bragg diffraction of plane (*H, K, L*).

Alternatively, one can abolish the small angle approximation while using the following sine function in Equation (8)

$$\sin\theta = \sin\left\{\frac{1}{2}\tan^{-1}\left(\frac{r}{D}\right)\right\}$$

Electron wavelength can be acquired by using Equation (3) or Equation (8) from which the crystal lattice constant or the Miller indices of a specific diffraction ring can be determined.

For face-centered cubic crystals such as gold and aluminum, geometric structure factors determine that only planes of all even-numbered or odd-numbered Miller indices can generate diffraction patterns, while other planes have zero reflection. Thus, Miller indices for generating diffraction rings are limited as shown in the table below.

$h \quad k \quad l$	$h^2 + k^2 + l^2$	$(h^2 + k^2 + l^2)^{\frac{1}{2}}$
111	3	1.732
200	4	2.000

220	8	2.828
311	11	3.316
222	12	3.464
400	16	4.000
331	19	4.358