3. Principle

3.1 Structure of CRT

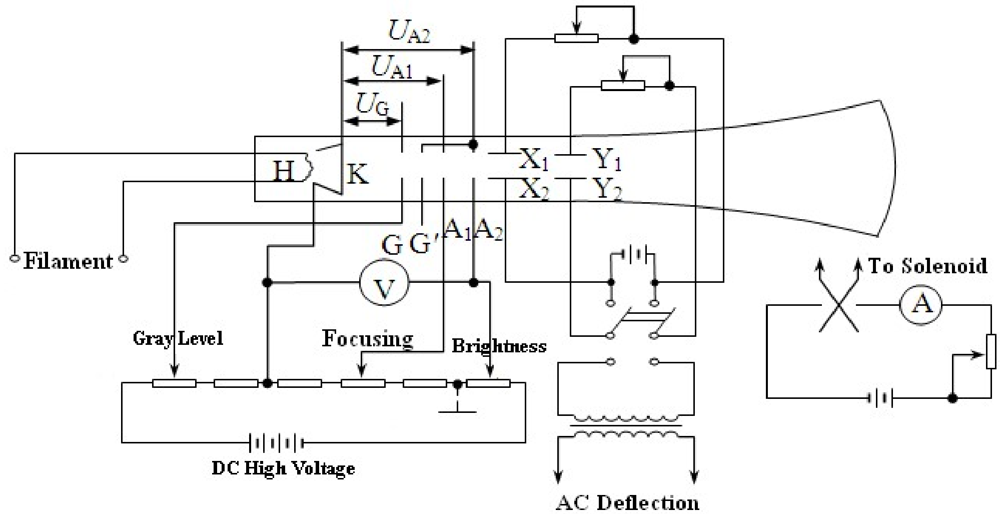

Figure 1 Schematic diagram of CRT

The main portion of the apparatus is a CRT, which consists of a glass shell, an electron gun, a set of deflection system, and a screen. The internal structure and working power supplies of a CRT are shown in Figure 1. After power up, filament *H* emits heat so that cathode *K* is heated and emits electrons. Next, a negative voltage of 5~20 V relative to cathode *K* is applied to grid *G* for two functions. One is to control the electron amount emitted by the cathode so that the grid electrode is also called the control electrode; while the other is to create a specific spatial potential distribution between grid *G* and electrode *G'* (*G'* has the same potential as the 2nd anode), making the electron beam emitted by the cathode form a cross point near grid *G* (i.e. a minimum interface), as shown in Figure 2.

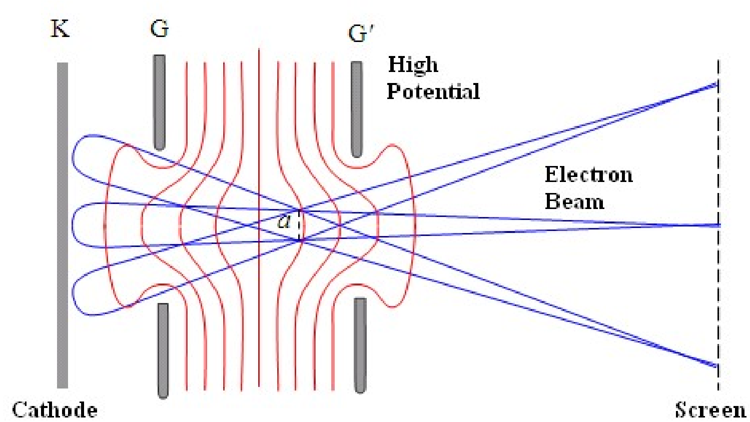

Figure 2 Schematic functions of grid *G*

Grid *G*, electrode *G'*, 1st anode A_1 and 2nd anode A_2 are all apertured cylindrical electrodes arranged in coaxial with cathode *K*. Electrode *G'* is connected with the 2nd anode A_2 with a working potential U_2 from a few hundred volts to a few thousand volts relative to cathode *K*. The 1st anode A_1 is applied with a potential U_1 of a few hundred volts relative to cathode *K* between U_K and U_2. On one hand, electrode G, 1st anode A_1 and 2nd anode A_2 create a focusing field, focusing the electron beam to a point; on the other hand, they accelerate electrons to hit and activate the fluorescence screen. The brightness on the fluorescence screen depends on the number and speed of electrons hitting the screen, which can be adjusted by changing potential across grid G or the accelerating voltage. The transversal and longitudinal deflection electrodes are two pairs of parallel metal plates placed perpendicularly to each other, which are used to control the position of the electron beam on the screen. A layer of conductive graphite is coated on the inner surface of the CRT, called shield electrode, which is connected with the 2nd anode allowing the secondary electrons generated by electron beam bombardment to flow backward thus avoiding charge accumulation near the screen. Finally, after passing through the 2nd anode, electrons move into an equipotential space.

3.2 Theory

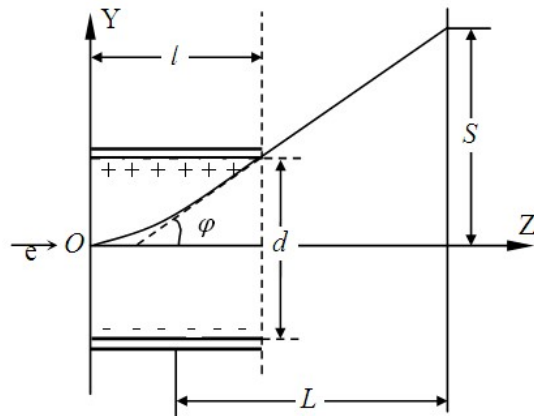

Figure 3 Principle of electric deflection

3.2.1 Electric deflection: electron beam in transversal electric field

The principle of electric deflection is seen in Figure 3. When a deflecting voltage V_d is applied to a pair of deflection electrodes of a CRT, electrons along Z-axis with velocity v are defected by the uniform electric field E in Y-axis, moving in a parabolic track. Outside the deflection electrode, electrons make a uniform rectilinear motion in the absence of an electric field.

Within the deflecting electrode, we have:

$$Y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}\left(\frac{Z}{v}\right)^2 = \frac{1}{2}\frac{eV_d}{md}\left(\frac{Z}{v}\right)^2 \tag{1}$$

where d is the separation of the deflection electrodes; e and m are the fundamental charge and mass of an electron, respectively; Y and Z are the displacements of electron beam in Y and Z axes, respectively.

If an electron is accelerated by a voltage of V_2, then we have

$$\frac{1}{2}mv^2 = eV_2 \tag{2}$$

By substituting (2) into (1), we get

$$Y = \frac{V_d Z^2}{4V_2 d} \tag{3}$$

The tangent of deflection angle φ of electron track relative to Z-axis is:

$$\mathrm{tg}\varphi = \left.\frac{\mathrm{d}Y}{\mathrm{d}Z}\right|_{Z=l} = \frac{V_d l}{2V_2 d} \tag{4}$$

where l is the length of the deflection electrodes in Z-direction. If the distance between the center of the deflection electrodes and the screen is L and the deflection displacement of the electron on screen is S, we have

$$tg\varphi = \frac{S}{L} \tag{5}$$

By substituting (5) into (4), we have

$$S = \frac{V_d l L}{2V_2 d} \tag{6}$$

It is apparent from (6) that deflection displacement S of electrons on the screen is proportional to deflection voltage V_d but inversely proportional to accelerating voltage V_2. Obviously, other parameters in (6) are those of the CRT, so (6) can be written as:

$$S = k_e \frac{V_d}{V_2} \tag{7}$$

where k_e is the electric deflection constant. Therefore, when accelerating voltage V_2 is fixed, the deflection displacement of electron beam is linearly proportional to the deflection voltage. The sensitivity of electric deflection in unit of mm/V is further defined as:

$$\delta_e = \frac{S}{V_d} = \frac{k_e}{V_2} \tag{8}$$

3.2.2 Electric focusing: electron beam in longitudinal electric field

Electric focusing is achieved with electrode G', 1st anode A_1 and 2nd anode A_2 to focus a spread electron beam onto a small spot on the screen. If an electron enters the accelerating field through cylindrical electrode G' with initial velocity v and angle θ with Z-axis, its trajectory is shown in figure 4. Since electron has a negative charge, electric force F exerting on electron is opposite to the direction of the electric field strength E, which is in the tangent direction of the electric field line as shown in figure 4. Thus, electrodes such as G', A_1, and A_2 are considered as electric lenses. Electric force F exerting on an electron can be further decomposed into two components, namely axial $F_{\parallel}$ (along z-axis) and radial $F_{\perp}$ (perpendicular to z-axis).

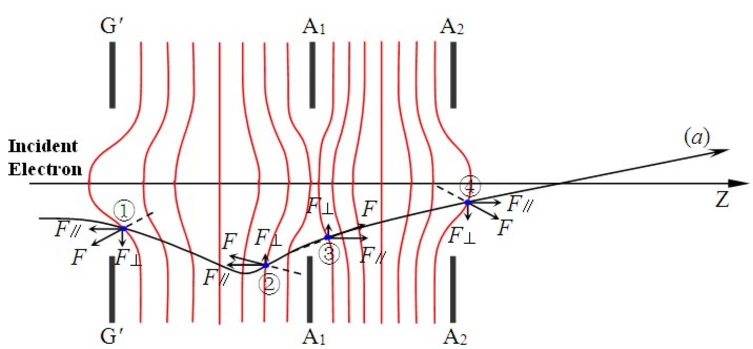

Figure 4 Schematic of electron trajectory by electric focusing

When the incident electron is at locations ①, ②, ③, and ④, it is subject to decelerate/ diverge, decelerate/converge, accelerate/converge, and accelerate/diverge, respectively. In electric fields, $F_{\parallel}$ accelerates electrons along z-axis direction while $F_{\perp}$ makes electrons bend to the z-axis in the first half of the lens range then deflect from z-axis in the second half of the lens range. As the electron is accelerated in the lens range, it stays in the first half of the lens range longer than the second half. The radial effect results in the trajectory of the electron bending to z-axis, so that the incident electron beam is focused.

As far as anodes A_1 and A_2 are concerned, it is their potential difference (U_{A1}-U_{A2}) rather than their individual voltage values (U_{A1}, U_{A2}) that determines the focusing function of an electron beam. So, the potential difference is equivalent to the focal length of the electric lens.

In a CRT, the 2nd anode is often called the accelerating electrode to accelerate electrons while the 1st anode is called the focusing electrode to change the ratio of U_{A1} to U_{A2} (the focal length of the electric lens). Of course, changing U_{A2} can also change the ratio of U_{A1} to U_{A2}, so the 2nd anode can play an auxiliary role in electric focusing.

Similar to the Gaussian imaging equation of geometrical optics, we get

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \tag{9}$$

where f, μ, and ν are the focal length, object distance, and image distance, respectively. Figure 5 shows the f~U_{A1}/U_{A2} curve of a CRT. It is apparent from figure 5 that no electric focusing is achieved in the absence of an electric field between anodes A_1 and A_2 when U_{A1}=U_{A2}, leading to an infinite focal length.

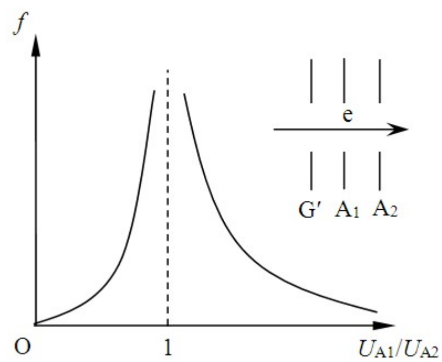

Figure 5 $f\sim U_{A1}/U_{A2}$ curve of CRT

3.2.3 Magnetic deflection: electron beam in transversal magnetic field

An electron moving at velocity v in a magnetic field is subject to a Lorenz force,

$$\boldsymbol{F}=-e\boldsymbol{v}\times\boldsymbol{B} \tag{10}$$

where B is the magnetic field strength. The direction of Lorenz force is always perpendicular to the direction of electron movement, thus changing the moving direction of the electron. Under such case, the centripetal force of the electron can be written as

$$evB=\frac{mv^2}{R} \tag{11}$$

where R is the radius of the circular trajectory trace of the electron in a transversal magnetic field. As shown in Figure 6, after the electron leaves the magnetic field region, it is subject to rectilinear motion in the absence of a magnetic force. Now, we have:

$$\sin\theta = \frac{l}{R} = \frac{leB}{mv} \tag{12}$$

where l is the range of the magnetic field, and θ is the deflecting angle. Since θ is small, $\sin\theta \cong \mathrm{tg}\theta$. Thus, deflection displacement s is:

$$s = L\,\mathrm{tg}\theta = \frac{leB}{mv}L \tag{13}$$

where L is the length of the CRT.

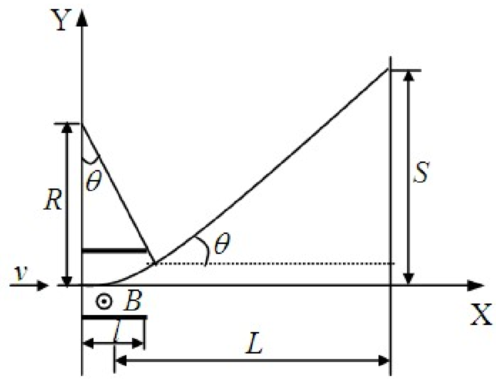

Figure 6 Electron deflection after leaving magnetic field

By substituting (2) into (13), we get

$$s = lBL\sqrt{\frac{e}{2mV_2}} \tag{14}$$

If the magnetic field is generated by a solenoid, then we get

$$B=\mu_0 nI=\mu_0 NI/L \tag{15}$$

where μ_0 is the permeability in vacuum, n is the number of turns of the coil per unit length, N is the number of turns of the coil, I is the current through the coil, and L is the axial length of the solenoid. Thus, the sensitivity of magnetic deflection is:

$$\frac{s}{I} = \mu_0 nlL\sqrt{\frac{e}{2mV_2}} \tag{16}$$

Unlike electric deflection seen in (6), magnetic deflection seen in (16) is inversely proportional to the square root of the accelerating voltage.

Magnetic deflection can be further used to measure the horizontal component of geomagnetic field. As shown in Figure 6, in the presence of a geomagnetic field, the distance between the deflected electron spot and the undeflected electron spot on screen is

$$D = R - R\cos\theta = R(1-\cos\theta) = \frac{mv}{eB}(1-\cos\theta) \tag{17}$$

Since θ is small, we get $\sin\theta \cong \theta$ and $\cos\theta \cong 1-\theta^2/2$. Thus, (17) can be rewritten as

$$D = \frac{mv}{eB}\frac{\theta^2}{2} = \frac{mv}{eB}\frac{\sin^2\theta}{2} \tag{18}$$

By substituting (12) and (2) into (18), we get:

$$D = \frac{l^2 eB}{2\sqrt{2meV_2}} \tag{19}$$

Here, l is the full length between the accelerating electrode and the screen.

To measure the horizontal component of geomagnetic field, first adjust accelerating voltage V_2 and focusing voltage to achieve a clear electron spot on the screen. Next, set both X and Y deflecting voltages to zero, and bring electron spot to horizontal axis while keeping V_2 unchanged. Then rotate the apparatus by 90°so that the horizontal component of geomagnetic field is now perpendicular to the electron beam. Under such condition, electrons are subject to the maximum deflection. Record the highest and lowest deflection displacements D_1 and D_2, and take the average of them as the deflection displacement of electron at accelerating voltage V_2. Finally, use formula (19) to calculate B for the horizontal component of geomagnetic field. **Note**: a compass may be needed to orientate the azimuth of the geomagnetic field.

3.2.4 Magnetic focusing and spiral motion: electron beam in longitudinal magnetic field

The magnetic focusing method known as the spiral focusing method can be used to measure the charge-mass ratio of an electron. Generally, when an electron enters a magnetic field, the velocity of the electron can be decomposed into two components: parallel component $v_{\parallel}$ and perpendicular component $v_{\perp}$ with respect to the direction of the magnetic field. The latter contributes to a Lorenz force as described previously making the electron undertake a circular motion in the plane perpendicular to the direction of the magnetic field; whereas the former is not affected by the magnetic field and, so simultaneously the electron is subject to a uniform rectilinear motion along the direction of the magnetic field. As a result, the trajectory of the electron is a spiral curve, as shown in Figure 7.

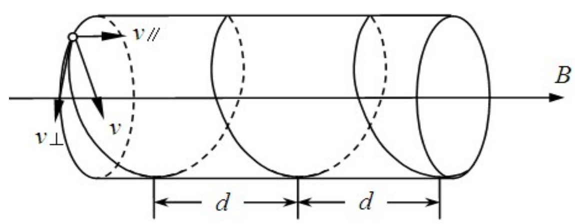

Figure 7 Spiral motion of electron in magnetic field

The pitch is

$$d = v_{\parallel} T = 2\pi m\, v_{\parallel} / eB \tag{20}$$

where T is the period of the electron moving one circle.

$$T = \frac{2\pi R}{v_{\perp}} \tag{21}$$

Also

$$ev_{\perp}B = \frac{m}{R}v_{\perp}{}^{2} \tag{22}$$

By substituting (21) and (22) into (20), we get:

$$e/m = 2\pi\, v_{\parallel} / Bd \tag{23}$$

As long as $v_{\parallel}$, d and B are measured, e/m can be calculated.

For a CRT as shown in Figure 1, we have

$$\frac{1}{2}m\, v_{\parallel}{}^{2} = e(U_{A2} - U_{K}) = eV_{2} \tag{24}$$

So the parallel velocity of the electron can be derived as

$$v_{\parallel} = \sqrt{\frac{2e(U_{A2} - U_{K})}{m}} = \sqrt{\frac{2eV_{2}}{m}} \tag{25}$$

By substituting (25) to (23), the charge-to-mass ratio of an electron can be derived as

$$\frac{e}{m} = \frac{8\pi^{2}V_{2}}{B^{2}d^{2}} \tag{26}$$

As shown in Figure 1, if a deflecting voltage is applied to Y_1 and Y_2, an electric field is created perpendicularly to the parallel direction of electron-beam, enabling the electron-beam to gain a perpendicular velocity component $v_{\perp}$ whereas the original parallel velocity component $v_{\parallel}$ of the electron remains unchanged. The perpendicular velocity component makes electron undertake a spiral motion.

If B is fixed, the perpendicular velocity of electron is a constant, leading to a constant period T; however, if an AC voltage is applied to Y_1 and Y_2, electrons passing through Y_1 and Y_2 during the positive half cycle (positive Y_1 and negative Y_2), gain different perpendicular velocities ($v_{\perp}$) undertaking spiral motion of different radii, as shown in Figure 8 (b). Under such condition, a straight line is observed on the screen. As shown in Figure 8 (a), initially, t_0=0 and $v_{\perp}$=0, so no Lorentz force is exerted on the electron; at time t_1, $v_{\perp}=v_{\perp 1}$, Lorentz force is f_1, the radius of the spiral trace is $R_1 = \frac{mv_{\perp 1}}{eB}$; at time t_2, $v_{\perp}=v_{\perp 2}$, Lorentz force f_2, the radius is $R_2 = \frac{mv_{\perp 2}}{eB}$. As a result, electrons make spiral motion of different radii on the right and left parts during the positive and negative half cycles, respectively.

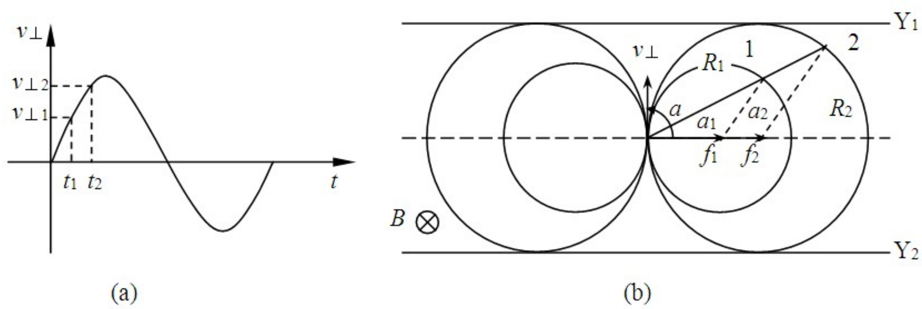

Figure 8 Spiral traces of electrons in AC magnetic field

For angular velocity, we have

$$\omega = \frac{v_\perp}{R} = \frac{eB}{m} \tag{27}$$

Obviously, ω is independent of $v_\perp$. If B is unchanged, the electrons from point "O" have the same angular velocity of spiral motion. If the distance from deflection electrode Y (point "O" as shown in Figure 1) to screen is L', since $v_\parallel$ is unchanged, all electrons emitted from "O" will take the same time $T_0=L'/v_\parallel$ to reach the screen. Hence, the rotation angles of electrons from point "O" to the screen are the same as $a_1=a_2=\omega T_0$ as shown in Figure 8(b). Therefore, bright spots "1" and "2" are on the same straight line passing through the axial center, but spot "1" reaches the screen ahead of spot "2" by $t_2 - t_1$. Due to the afterglow effect, spot "1" does not disappear before spot "2" appears. Similarly, other electrons from point "O" hit on the same straight line on the screen and therefore a bright straight line is observed on the screen.

If B is increased, it is apparent from (27) that the radius of the spiral motion decreases, so that the bright line is shortened; in the meantime, ω increases with an increase in B, so that the straight line also rotates while shortening as shown in Figure 9.

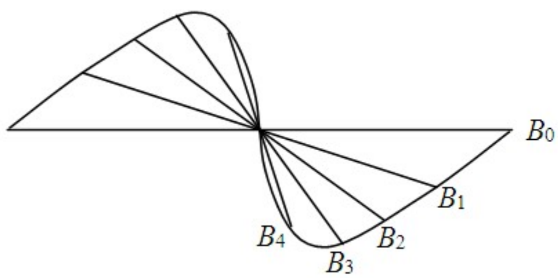

Figure 9 Change pattern of bright line on screen

The strength of B can be changed to make the rotation angle equal to 2π in a complete circle of the spiral motion. Under such condition, Thus, electrons emitted from "O" return to the axis after a complete circle of spiral motion but move forward with a distance d (pitch). As a result, a bright spot is seen on the screen, called the 1st focusing spot ($d=L'$, where L' is the distance between the deflection electrode and the screen). If B is increased continuously, the 2nd, 3rd, … focusing spots can be seen on the screen with $d=L'/2$, $d=L'/3$, ..., respectively.

As shown in Figure 10, the magnetic strength B at a specific position on the axis in a solenoid can be rewritten from (15) to

$$B = \frac{\mu_0}{2} nI(\cos B_2 - \cos B_1) \tag{28}$$

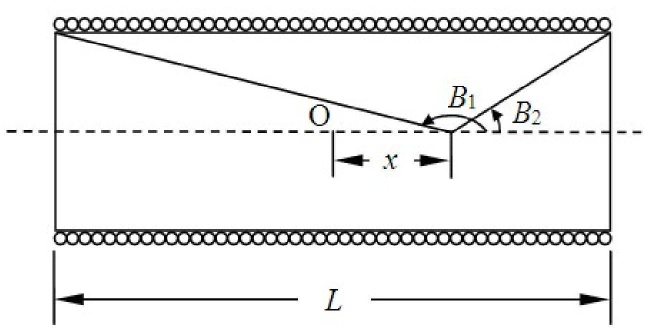

Figure 10 Schematic of solenoid

Further, (28) can be rewritten as

$$B = \frac{\mu_0}{2} nI \left[\frac{\frac{L}{2} - x}{\sqrt{(\frac{D}{2})^2 + (\frac{L}{2} - x)^2}} + \frac{\frac{L}{2} + x}{\sqrt{(\frac{D}{2})^2 + (\frac{L}{2} + x)^2}} \right] \tag{29}$$

where L and D are the length and diameter of the solenoid, respectively; x is the displacement of the location on the central axis from the middle location of the axis. It is apparent from (28) that B is a nonlinear function of x. However, if L is large enough and only the middle portion of the solenoid is used, the magnetic field of the solenoid can be approximated with a uniform magnetic field described by (15). As otherwise, (15) should be modified with a correction coefficient K

$$\overline{B} = K \mu_0 nI = K\mu_0 NI/L \tag{30}$$

$$K = \frac{1}{2x_0} \left[\sqrt{(\frac{D}{2})^2 + (\frac{L}{2} + x_0)^2} - \sqrt{(\frac{D}{2})^2 + (\frac{L}{2} - x_0)^2} \right]$$

where x_0 is the displacement of the end point of a finite solenoid on the central axis from the middle point of the axis ($x_0 \cong L/2$). In this experiment, the polar coordinate equation of electron spiral motion can be determined. When the electron moves by distance L along the axial direction, the total angle φ it rotates is:

$$\varphi = \omega L / v_{\parallel} = eBL / m v_{\parallel} = \frac{2\pi L}{d} \tag{31}$$

Since both φ and L can be directly measured, (31) can be used to calculate d which cannot be directly measured in the experiment. As shown in Figure 11, point A is the position of electron hitting the screen (bright spot), origin O is the bright spot position when $v_{\perp} = 0$, and R is the radius of the circle of the spiral motion. By changing B while keeping $v_{\perp}$ unchanged, electron will undertake a spiral motion with the corresponding polar coordinates described as:

$$r = 2R \sin \frac{\varphi}{2} \tag{32}$$

$$\theta = \frac{\varphi}{2}$$

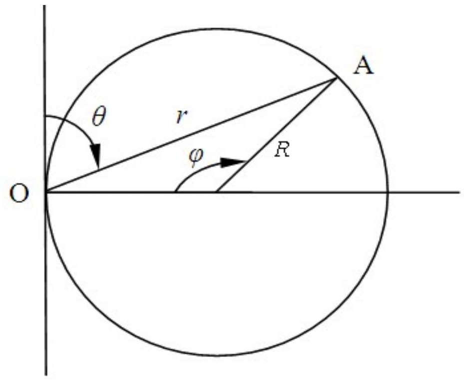

Figure 11 Schematic of polar coordinates

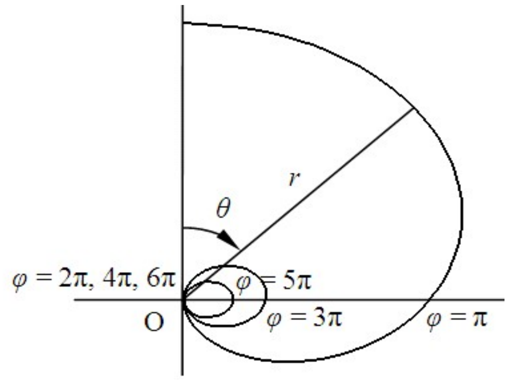

Figure 12 Vortex curve

<u>Note</u>: φ should be added by an integer times of 2π if the electron rotating more than one circle.

Since R and φ are related to B, so are r and θ. From (27), (31), and (32), we get

$$r = \frac{v_{\perp} L \sin(\theta)}{v_{//} \theta} \tag{33}$$

(33) is the equation of a vortex curve as shown in Figure 12.

<u>Note</u>: as shown in Figure 12, when θ takes an integer of π, the corresponding φ is the integer of 2π, i.e. spiral motion rotates integer circles, r becomes zero, electron beam returns to the un-deflected position, bright spot position coincides with "O". As B increases, so does φ, but the amplitude of electron beam deflection decreases. When φ is an odd number of π, the bright spot position will be on the X axis.