3. Working principle

After switching on, the electron in the oscilloscope tube will be subject to Lorentz force under the uniform magnetic field generated by the coil. The Lorentz force can be expressed as:

$$\vec{F} = e\vec{v} \times \vec{B} \tag{1}$$

where $\vec{F}$ is the Lorentz force exerted on the electron, $\vec{v}$ is the velocity of electron movement, $\vec{B}$ is the magnetic induction strength, and *e* is the electron charge.

The direction of Lorentz force is decided by left hand rule and its magnitude is:

$$F = evB\sin\alpha \tag{2}$$

where α is the intersection angle between the velocity vector of electron and the magnetic field vector. If the electron's moving direction coincides with the field direction, then a=0, sin$a$=0. The force exerted on the electron is also zero, and thus the electron will keep its uniform rectilinear movement along its original directions.

When the electron velocity $\vec{v}$ is vertical to $\vec{B}$ (electron velocity is expressed as $v_{\uparrow}$),a=90°,sin$a$=1. Under such case, the force exerted onthe electron becomes$F=evB$. This force changes electron's moving direction, but it doesn't change the magnitude of the electron velocity, and hence the electron makes a uniform circular movement with a radius *R* in the plane vertical to *B*, as shown in Figure 1(a).

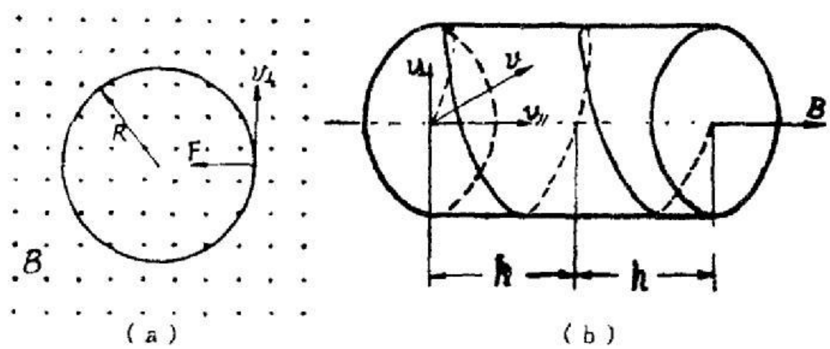

Figure 1 Moving trace of electron in magnetic field

From Newton's law, we get:

$$F = ev_{\uparrow}B = \frac{mv_{\uparrow}^{2}}{R} \tag{3}$$

Here, *m* is the mass of an electron.

Thus, the time necessary for an electron to rotate for one revolution is:

$$T = \frac{2\pi R}{v_{\uparrow}} = \frac{2\pi m}{eB} \tag{4}$$

Equation (4) shows that period T is independent of electron velocity, indicating that, under a uniform magnetic field, the time of rotation of one revolution for electrons with different velocities is identical, except that the higher the electron velocity, the larger the rotation radius of the electron. This is actually the theoretical basis for the magnetic focusing method.

If there is an anglebetween electron velocity v and magnetic field B, the electron velocity can be decomposed into two components: axial velocity $v_{//}$ parallel with B; radial velocity $v_{\uparrow}$ vertical to B. $v_{//}$ remains constant, i.e. the electron will uniformly move along the axis; while under Lorentz force, $v_{\uparrow}$ will make the electron rotate around the axis. Theend result is thatthe orbit of movement is a helix as shown in Figure 1 (b), and the pitch (the travel of the electron in B direction between revolutions) is:

$$h = Tv_{//} = \frac{2\pi m}{eB} v_{//} \tag{5}$$

From equation (5), we get:

$$\frac{e}{m} = \frac{2\pi}{Bh} v_{//} \tag{6}$$

Here, h is the pitch.

If the electrons in an electron beam emitted from the same point have different $v_{\uparrow}$ but with identical $v_{//}$, then after travelling a distance h, they will be focused at a point. This is called magnetic field focusing (i.e. longitudinal magnetic field focusing).
The electrons emitted from a cathode can be approximately assumed to have no initial velocity. They will be accelerated under the voltage of the first anode (focusing electrode) and second anode (accelerating electrode) in the tube. The longitudinal velocity of electron $v_{//}$ depends on voltage U (referred to as acceleration voltage) applied by the cathode to the second anode, i.e. $\frac{1}{2} m v_{//}^{2} = eU$.

Hence, we get:

$$v_{//} = \sqrt{\frac{2eU}{m}} \tag{7}$$

Substituting equation (7) into equation (6), we get:

$$\frac{e}{m} = \frac{8\pi^2 U}{h^2 B^2} \tag{8}$$

The oscilloscope tube is placed with the long helix tube. After switching on, all the electron rays will be focused into a point under the focusing voltage, thereby a bright spot will be found on the screen. To create velocity $v_{\uparrow}$, 15 VAC is applied to a pair of deflection plates of the oscilloscope tube and the electrons will get a vertical component velocity within a certain range and thus a scanning straight line is formed on the screen.

If current $I$is applied tothe helix tube, a magnetic induction strength will be generated in the direction of this helix tube. Under the magnetic field, electrons will make a helix movement, It is apparent from Equation (5) that $v_{//}$ (also acceleration voltage U) varies with$I$ (magnitude of B) in the helix tube, in such a way that h (pitch) is just equal to the distance(l) between the starting deflecting point of the y deflection plate and the center of the screen. Under such case, a bright spot shown on the screen is called primary focusing as seenin Figure 2. Thus, equation (8) can be rewritten as:

$$\frac{e}{m} = \frac{8\pi^2 U}{l^2 B^2} \qquad (l \approx 0.148\ m) \tag{9}$$

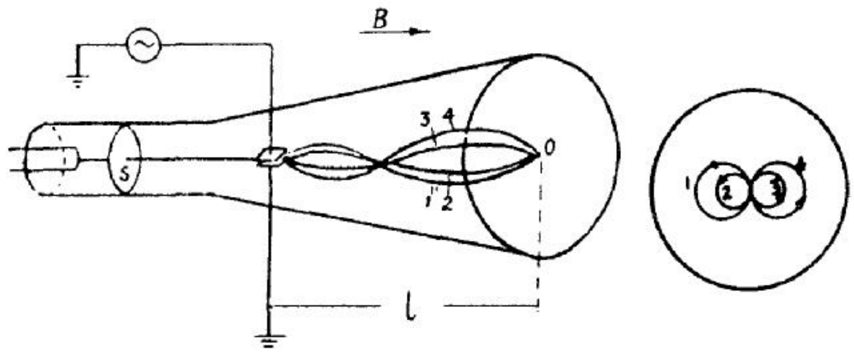

Figure 2 Moving traces of electrons in helix tube

In fact, the magnetic field in a helix tube should be calculated based on field formula for a multi-layer wound helix tube. However, it can be simplified for a thin helix tube as seen in Figure 3:

$$B = \frac{\mu_0 NI(\cos\beta_1 - \cos\beta_2)}{2} \tag{10}$$

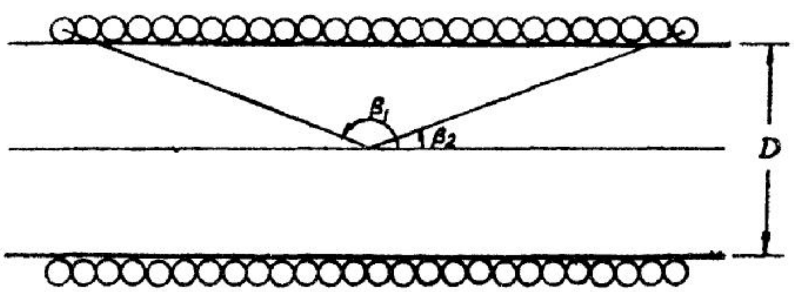

Figure 3 Schematic of thin helix tube

At the center of the helix tube, we get $\beta_1+\beta_2=180°$, so Equation (10) becomes

$$B = \mu_0 NI \cos\beta_2 \tag{11}$$

Hence, the equation of e/m ratio is:

$$\frac{e}{m} = \frac{8\pi^2 U}{l^2 \mu_0{}^2 N^2 I^2 \cos^2 \beta_2} = \frac{U}{2l^2 N^2 I^2 \cos^2 \beta_2} \times 10^{14} \text{ C/kg} \quad (12)$$

where$N$:number of turns per unit length (turns/m) (for this apparatus N=3800 turns/m); μ_0: the magnetic conductivity in vacuum ($\mu_0=4\pi\times10^{-7}$H/m);I: the current in helix tube;l: the distance between screen and y deflection plate;U: acceleration voltage; and $\cos\beta_2=0.948$.