5. Theory

If an oil drop with mass *m* and charge *e* enters the oil chamber, it falls down freely under the force of gravity when no voltage is applied between the parallel polar plates. When the force of gravity is balanced against air resistance (neglecting air buoyancy), the oil drop falls down at a uniform speed of V_g, as described by the following equation:

$$mg = f_a \tag{1}$$

where f_a is the air resistance when the oil drop falls down at a uniform speed of V_g, and g is the gravitational constant. According to Stokes' law, the air resistance exerted on the oil drop is

$$f_a = 6\pi\eta r V_g \tag{2}$$

While the force of gravity exerted on the oil drop is

$$G = mg = \frac{4}{3}\pi r^3 \rho g \tag{3}$$

where η is the coefficient of viscosity for air; ρ is the density of oil drop; and r is the radius of the oil drop. By combining equations (1) to (3), the radius of the oil drop can be derived as:

$$r = 3\sqrt{\frac{\eta V_g}{2g\rho}} \tag{4}$$

When an electric field is applied across the parallel polar plates, the oil drop is subject to the force of the electric field. If the strength and polarity of the electric field are controlled and selected properly, the oil drop can elevate upward under the influence of the electric field. If the force of the electric field exerted on the oil drop is balanced against the sum of the force of gravity and the resistance of air exerted on the oil drop, the oil drop will elevate upward at a uniform speed of V_s.

Hence, we have

$$mg + f_a^{'} = QE \tag{5}$$

where E is the strength of the electric field applied to the parallel polar plates and Q is the electric charge on the oil drop.

From equations (1) and (2), equation (3) can be rewritten as

$$mg + f_a^{'} = f_a + f_a^{'} = 6\pi\eta r(V_g + V_s) = QE = Q\frac{U}{d} \tag{6}$$

Hence,

$$Q = \frac{6\pi\eta rd(V_g + V_s)}{U} \tag{7}$$

where U and d are the voltage and separation between the parallel polar plates, respectively. Equation (7) is derived assuming the oil drop is in a continuous medium. In this experiment, the radius of the oil drop is as small as 10^{-6} m and therefore it is comparable to air molecules. As a result, air is no longer considered as a continuous medium. Therefore, equation (7) needs to be modified to

$$Q = \frac{6\pi\eta rd}{U}(V_g + V_s) / \left(1 + \frac{8.12 \times 10^{-8}}{Pr}\right)^{\frac{3}{2}} \tag{8}$$

where P is the atmospheric pressure in unit *atm* (approximately P=1*atm*). In this experiment, the coefficient of viscosity for air is η=1.83×10^{-5} kg/ms; the density of the oil drop is ρ=981 kg/m^3; and the distance between the parallel polar plates is d=5×10^{-3} m.

Therefore, V_g, V_s, and U should be measured first; then the radius of the oil drop r can be calculated from equation (4); finally, the quantity of electric charge on the oil drop can be found from equation (8). Many oil drops have been measured with the above method and the results shown the charge quantities carried by oil drops are always multiples of a certain minimum value. This minimum fixed value of charges is the elementary charge as defined as e≅1.6021892×10^{-19} coulombs.

When processing the experimental data, students can verify the data in a reverse order: divide the experimental data (charge quantity Q) by the well-recognized electric elementary charge value e≅1.6021892×10^{-19} C. The obtained quotient is a value close to a certain integer, which is the charge quantity n carried by the oil drop. By dividing Q by n, the obtained quotient is the experimental value of the elementary charge.