2. Theory

1) Theory

In the study of the mean free path of very low-energy electron (0.75 eV~1.1 eV), Ramsauer found the electron free path of argon gas was much greater than the calculated value using gas molecule kinetic theory. By extending the electronic energy to a wide range, he found the total effective cross section Q for elastic scattering of electron with argon atom increases as the electron energy decreases. In the vicinity of 10 eV, Q becomes maximal, and then begins to decline. When the electron energy is gradually reduced to 1 eV approximately, Q becomes minimal. In other words, for electron energy of about 1 eV, argon gas looks like transparent. Q increases again after the electron energy is less than 1 eV. Since then, Ramsauer measured various gases, and found the total effective scattering cross sections of all gases are related to the speed of electrons. For structurally similar gas atoms or molecules, their relationship curves $Q=F(V^{1/2})$ (V is the acceleration voltage) have the same shape, known as Ramsauer curve seen in Figure 1.

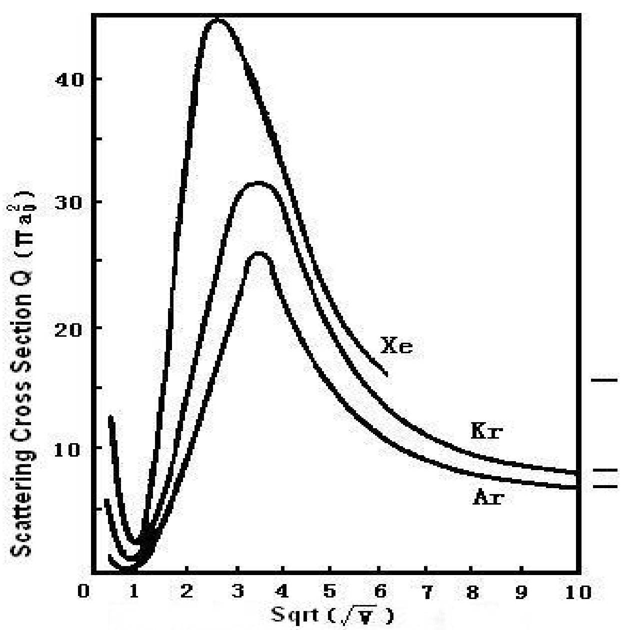

Figure 1 Ramsauer curves of gases of Xe, Ke and Ar

Figure 1 shows the Ramsauer curves of three inert gases: Xenon (Xe), Krypton (Ke) and Argon (Ar), where the horizontal coordinate is the square root of the acceleration voltage which is proportional to the speed of electrons; the vertical coordinate is the scattering cross section Q in atomic unit (AU) with a_0 being the Bohr radius. The short horizontal lines on the right of the frame represent the calculated values of Q based on gas kinetic theory. Obviously, it cannot explain Ramsauer effect if a collision model of two rigid balls is used to explain the interaction between electron and atom, as the scattering cross section would be independent of the electron energy. To explain Ramsauer effect, the wave nature of particles should be used, namely, the collision between electron and atom should be considered as scattering of incident particles in the atomic potential field and the degree of scattering is represented by the total scattering cross section.

A qualitative interpretation about Ramsauer-Townsen effect using quantum mechanics is given as follows: if ψ is the wave function of the electron and $V(r)$ is the interaction potential between electron and atom, we get:

$$\psi \xrightarrow[r\to\infty]{} e^{ikz} + f(\theta)\frac{e^{ikz}}{r} (k = \sqrt{2mE/h^2}) \tag{1}$$

where m is the mass of an electron, h is the Planck constant.

By solving the Schrödinger equation

$$\left[-\frac{h^2}{2m}\nabla^2 + V(r)\right]\psi = E\psi \tag{2}$$

A theoretical curve can be acquired. For Xenon, Krypton, and Argon atoms, the minimum scattering cross section is in the vicinity of 1 eV.

For $V(r)$, the most simplified model is the one-dimensional square potential well. For a given potential well of V_0, if the energy of the incident particle meets conditions of:

$$k'a = n\pi$$
$$k' = \sqrt{2m(E+V_0)/h^2} = 2\pi/\lambda \tag{3}$$

where a is the width of the potential well and n is an integer. In other words, if the width of the potential well is an integer multiple of one half of the wavelength of incident particle, resonance transmission occurs.

According to this model, the scattering cross section should periodically appear minima as the electron energy changes. However, as shown in Figure 1, only one minimum occurs in the vicinity of 1 eV. By considering the potential field of inert gases as a three-dimensional square potential well, the shape of Ramsauer curve can be qualitatively explained as follows:

$$V(r) = \begin{cases} -V_0 & (r < a) \\ 0 & (r > a) \end{cases} \tag{4}$$

Since $V(r)$ is only related to the relative position between electron and atom but independent of the angle, $V(r)$ is a central force field whose wave function can be expressed as the coherent superposition of incident wave and emitting wave with different angular momentum l. For each l– referred to a partial wave, the role of the central force field $V(r)$ is to create a phase shift in the radial direction. The total scattering cross section is:

$$Q = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin\delta_l \tag{5}$$

Now, to calculate the total scattering cross section, one needs to calculate the phase shift δ_l of each partial wave that can be acquired by solving equation in radial direction:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}R_l\right) + \left[k^2 - \frac{l(l+1)}{r^2} - U(r)\right]R_l = 0 \tag{6}$$

Thus we get

$$R_l \xrightarrow{kr\to\infty} \frac{1}{kr}\sin\left(kr - \frac{l\pi}{2} + \delta_l\right) \tag{7}$$

where

$$k^2 = 2mE/\hbar^2,\quad U(r) = 2mV(r)/\hbar^2,\quad l = 0,\ 1,\ 2,\ \ldots \tag{8}$$

For the case of low energy, i.e. $ka << 1$, the contribution of high-order partial waves is small, and hence only the phase shift δ_0 of the zero-order (l=0) partial wave needs to be calculated. So, equation (5) becomes:

$$Q_0 = \frac{4\pi}{k^2}\sin^2\delta_0 \tag{9}$$

If $k\neq 0$, but $\delta_0=\pi$, we get $Q_0=0$. That means when the partial wave (l=0) crosses zero while the cross sections Q_1, Q_2, … of high-order partial waves are very small, the total scattering cross section can become minimal. On the other hand, by solving equation (6) when l=0, the condition of $\delta_0=\pi$ can be obtained as

$$tg(k'a) \approx k'a \tag{10}$$

where $k' = \sqrt{2m(E+V_0)/\hbar^2}$.

By adjusting potential well parameters V_0 and a, scattering cross section can become minimal when the energy of incident particle is 1eV, i.e. resonant transmission occurs. When the energy gradually increases, the contribution of high-order partial waves cannot be neglected−equation (6) of $l\neq 0$ needs to be solved. Unlike the one-dimensional case, the total cross section will no longer display periodical decreases and the model of three-dimensional square potential can be used to explain Ramsauer curve more precisely. Thus, by analyzing the relationship between elastic scattering cross section and electron energy, we can interpret the atomic potential field.

2) Principle of Measurement

The schematic of a Xenon gas filled electron collision tube is shown in Figure 2. The box-like screen electrode *S* of the tube is separated into left and right regions in the middle by a partition plate with an open rectangular hole. In the left region, there is a cylindrical-shape heated oxide cathode *K* with a built-in spiral filament *H* (heater). There is a grid *G* between the cathode and the partition plate. The right region is the equal-potential area. Electrons pass through the open hole of the partition plate collide elastically with Xenon atoms in this region. Plate *P* located in this region collects the transmission electrons that are not scattered.

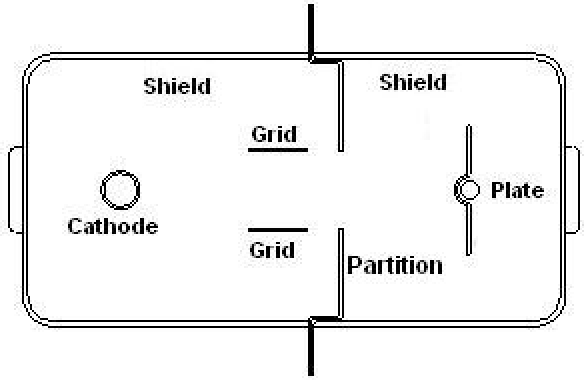

Figure 2 Schematic of Xenon gas filled electron collision tube

Figure 3 shows the schematic diagram for the measurement of the total scattering section of gas atoms. When the filament is heated, electrons escape from the cathode and are accelerated by acceleration voltage. Before reaching the grid, one portion of the electrons is received by the wall of the shield generating current I_{s1} while the rest electrons pass through the rectangular hole of the partition plate, generating current I_0. Since there is an equal potential space between plate *P* and the partition plate, electrons passing through the open hole of the partition plate will travel at a constant speed. The electrons that are scattered by gas atoms will reach the wall of shield *S* forming scattering current I_{s2}, whereas the electrons that are not scattered will reach plate *P* forming plate current I_p.

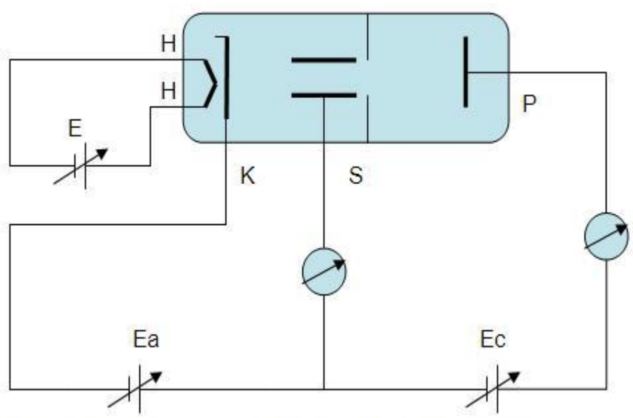

Figure 3 Schematic diagram for measurement of the total scattering section of gas atom

Thus, we get

$$I_K = I_0 + I_{S1} \tag{11}$$

$$I_S = I_{S1} + I_{S2} \tag{12}$$

$$I_0 = I_P + I_{S2} \tag{13}$$

where I_k is the cathode current.

The scattering probability of electrons in the equal potential region is:

$$P_S = 1 - \frac{I_P}{I_0} \tag{14}$$

Thus, scattering probability can be derived by measuring I_p and I_0. From the above discussion, I_p can be measured directly while I_0 has to be measured indirectly. Since cathode current I_k can be divided into I_{s1} and I_0, which are not proportional to I_k. Furthermore, there exists a certain proportional relationship between I_{s1} and I_0, called a geometric factor f as shown below.

$$f = \frac{I_0}{I_{S1}} \tag{15}$$

Factor f is determined by the relative angle between electrodes and the space charge effect. In other words, f is related to the geometry of the tube, accelerating voltage and cathode current. By substituting (15) into (14), we get:

$$P_S = 1 - \frac{1}{f}\frac{I_P}{I_{S1}} \tag{16}$$

To measure geometry factor f, the end portion of the electron collision tube is immersed into liquid Nitrogen at 77 K. Thus, the gas in the tube is frozen and the density of the gas atoms is very low. Under such case, the scattering between electrons and gas atoms can be ignored and geometry factor f is then equal to the ratio between plate current I_p^* and shield current I_s^* as

$$f \approx \frac{I_P^*}{I_S^*} \tag{17}$$

Thus, equation (16) can be rewritten as

$$P_S = 1 - \frac{I_P}{I_{S1}}\frac{I_S^*}{I_P^*} \tag{18}$$

From equations (12) and (13), we get:

$$I_S + I_P = I_{S1} + I_0 \tag{19}$$

From equations (15) and (17), we have:

$$I_0 = I_{S1}\frac{I_P^*}{I_S^*} \tag{20}$$

Thus, we have:

$$I_{S1} = \frac{I_S^*(I_S + I_P)}{(I_S^* + I_P^*)} \tag{21}$$

By substituting (21) into (18), we get

$$P_S = 1 - \frac{I_P}{I_P^*}\frac{(I_S^* + I_P^*)}{(I_S + I_P)} \tag{22}$$

Equation (22) is the formula used to measure the scattering probability in this experiment.
The total effective scattering cross section of electrons, Q, and the scattering probability follow a simple relationship as:

$$P_S = 1 - \exp(-QL) \tag{23}$$

where L is the distance between partition plate S and plate P.

From equations (22) and (23), we have:

$$QL = \ln\left(\frac{I_P^*(I_S + I_P)}{I_P(I_S^* + I_P^*)}\right) \tag{24}$$

Since L is a constant, by acquiring the relationship curve between $\ln\left(\frac{I_P^*(I_S + I_P)}{I_P(I_S^* + I_P^*)}\right)$ and $\sqrt{E_c}$, the relationship between the total effective scattering cross section of electrons and the speed of electrons can be achieved.