

2. Theory

A. Faraday Effect

As seen in Figure 1, if a magnetic field is not very strong, the rotation angle of the polarization direction of light transmitted through a medium under the magnetic field is proportional to the product of path length d of the light propagating in the medium and magnetic field strength B along the propagation direction in the medium, i.e. $\theta = V B d$, here coefficient V depends on the medium and the wavelength of the light. It is a representation of the magneto-optic properties of a material, called the Verdet constant.

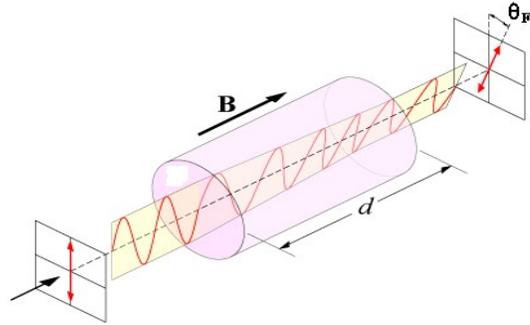


Figure 1 Schematic of Faraday effect

For paramagnetic, weak magnetic and diamagnetic materials (such as heavy flint glass), V is a constant. Therefore, rotation angle θ has a linear relationship with the magnetic field intensity B ; while for ferromagnetic materials (such as YIG cubic crystal material), θ and B do not have a simple linear relationship.

Table 1 lists the Verdet constants of some substances.

Table 1 Verdet constants of common materials (in unit of $\text{rad} \cdot \text{Tesla}^{-1} \cdot \text{cm}^{-1}$)

Material	λ (nm)	V
Water	589.3	0.038
CS ₂	589.3	0.121
Light flint glass	589.3	0.092
Heavy flint glass	830.0	0.233~0.291
Crown glass	632.8	0.061~0.212
Acrylic	632.8	0.0263
Quartz	632.8	0.141
Phosphorus	589.3	0.358
MR3-2 magneto-optic glass	632.8	-0.959

Different substances may differ in the direction of the polarization plane rotation. When observing along the magnetic field, if the rotation direction satisfies the right hand rule, it is

called “right rotation” medium and its Verdet constant is positive ($V>0$); while the reverse rotation is known as “left rotation” medium and its Verdet constant is negative ($V<0$).

For a given substance, Faraday rotation direction solely depends on the magnetic field direction. This differs from the natural optical activity effect of some substances, which is related to the light propagation direction. As a result, the rotation angle of the Faraday effect will double if light passes through the medium in a round trip. Similar to the inherent optical activity effect, the Faraday effect is also subject to optical rotatory dispersion, i.e. the Verdet constant is a function of wavelength.

B. Zeeman Effect

1) Relationship between total magnetic moment and total angular momentum

Strictly speaking, the total magnetic moment of an atom consists of electron magnetic moment and nuclear magnetic moment while the former is three orders of magnitude larger than the latter. For this reason, only the electron magnetic moment is considered here. The orbital motion of an electron in an atom creates orbital magnetic moment, while the spin motion of an electron results in spin magnetic moment.

Based on quantum mechanics, the numerical relationship between the orbital magnetic moment μ_L and the orbital angular momentum P_L of an electron is as follows:

$$\mu_L = \frac{e}{2m} P_L \quad \text{with } P_L = \sqrt{L(L+1)}\hbar \quad (1)$$

The relationship between the spin magnetic moment μ_S and the spin angular momentum P_S is:

$$\mu_S = \frac{e}{m} P_S \quad \text{with } P_S = \sqrt{S(S+1)}\hbar \quad (2)$$

where e and m are the charge and mass of an electron, respectively; L and S are the orbital quantum number and spin quantum number, respectively. The total angular momentum of the atom, P_J , is the sum of orbital angular momentum and spin angular momentum; while the total magnetic moment μ is the sum of orbital magnetic moment and spin magnetic moment. Since μ moves around P_J , the net projection of μ on P_J is not zero ($\mu_J \neq 0$). The numerical relationship between μ_J and P_J is written as:

$$\mu_J = g \frac{e}{2m} P_J \quad (3)$$

where

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} \quad (4)$$

is the Lande g -factor that determines the energy-level splitting amount in a magnetic field.

2) Effect of external magnetic field on atomic energy levels

In an external magnetic field, the introduced torque L on the total magnetic moment μ of an atom is:

$$L = \mu_J \times B \quad (5)$$

where B is the magnetic induction. Torque L forces angular momentum \mathbf{P}_J moving around the magnetic field direction. This motion brings additional energy:

$$\Delta E = -\mu_J B \cos \alpha \quad (6)$$

Substituting (3) into (6), we get:

$$\Delta E = g \frac{e}{2m} P_J B \cos \alpha \quad (7)$$

Since the orientations of μ_J and \mathbf{P}_J are quantized in a magnetic field, i.e. the components of \mathbf{P}_J are quantized in the direction of the magnetic field, and it must be an integer times of \hbar as,

$$P_J \cos \alpha = M\hbar \quad M = J, (J-1), \dots, -J \quad (8)$$

There are totally $2J+1$ magnetic quantum numbers. Substituting (8) into (7), we get:

$$\Delta E = Mg \frac{e\hbar}{2m} B \quad (9)$$

Thus, one energy level is split into $2J+1$ sub-levels in an external magnetic field with energy difference between sub-levels proportional to the external magnetic field B and the Lande g -factor, as determined by Eq. (9).

3) Selection rules of Zeeman effect

If a spectral line is emitted by electron transition from energy level E_2 to energy level E_1 in the absence of an external magnetic field, frequency ν of the spectral line is given by

$$h\nu = E_2 - E_1 \quad (10)$$

In the presence of an external magnetic field, the upper and lower energy levels are split into $2J_2+1$ and $2J_1+1$ sub energy levels with additional energies ΔE_2 and ΔE_1 , respectively. The frequency of the new spectral line ν' is given by:

$$h\nu' = (E_2 + \Delta E_2) - (E_1 + \Delta E_1) \quad (11)$$

Therefore, the frequency difference between the spectral lines is:

$$\Delta\nu = \nu' - \nu = \frac{1}{h}(\Delta E_2 - \Delta E_1) = (M_2 g_2 - M_1 g_1) \frac{eB}{4\pi m} \quad (12)$$

If represented by wave number, Eq. (12) can be rewritten as:

$$\Delta\tilde{\nu} = (M_2 g_2 - M_1 g_1) \frac{eB}{4\pi mc} \quad (13)$$

The Lorentz unit $L=eB/(4\pi mc)=4.67 \times 10^{-3} \text{ Bm}^{-1}$, where B is in unit of Gs ($1 \text{ Gs}=10^{-4} \text{ T}$).

There exist transition selection rules that must be met as: $\Delta M=M_2-M_1=0, \pm 1$ with an exception of $M_2=0 \rightarrow M_1=0$ when $J_2=J_1$.

(1) when $\Delta M=0$, π lines are generated with linear polarization parallel to the magnetic field when observing along the direction perpendicular to the magnetic field. When observing along the magnetic field, light intensity is zero.

(2) when $\Delta M=\pm 1$, σ^\pm lines are generated (called σ lines) with linear polarization perpendicular to the magnetic field when observing along the direction perpendicular to the magnetic field. When the propagation direction of light is along the direction of the magnetic field, σ^+ line is left-handed circularly polarized while σ^- line is right-handed circularly polarized; when the propagation direction of light is 180° of the direction of magnetic field, the observed σ^+ and σ^- lines are right-handed and left-handed circularly polarized, respectively; when observed in other directions, π lines still remain as linearly polarized, but σ lines are circularly polarized.

4) Zeeman effect of Mercury green line

The Mercury green line used in this Zeeman experiment is at 546.1 nm between energy levels $6s7s^3S_1 \rightarrow 6s6p^3P_2$. The two energy levels with the corresponding quantum numbers, g , M , Mg and polarization, are listed in Tables 2 and 3.

Table 2 Polarization of spectral lines

Selection rules	$K \perp B$ (transverse)	$K \parallel B$ (longitudinal)
$\Delta M=0$	Linearly polarized π component	No light
$\Delta M=+1$	Linearly polarized σ component	right-handed circularly polarized
$\Delta M=-1$	Linearly polarized σ component	left-handed circularly polarized

where K is the optical wave vector, B is the magnetic induction vector, σ represents vector $E \perp B$, and π represents vector $E \parallel B$.

Table 3 Quantum numbers of Mercury energy states

Atomic states	7^3S_1	6^3P_2
L	0	1
S	1	1
J	1	2
g	2	3/2
M	1, 0, -1	2, 1, 0, -1, -2
Mg	2, 0, -2	3, 3/2, 0, -3/2, -3

The Lande factor g and the splitting of the two atomic states in a magnetic field can be calculated by (4) and (7), with the transition diagram plotted in Figure 2.

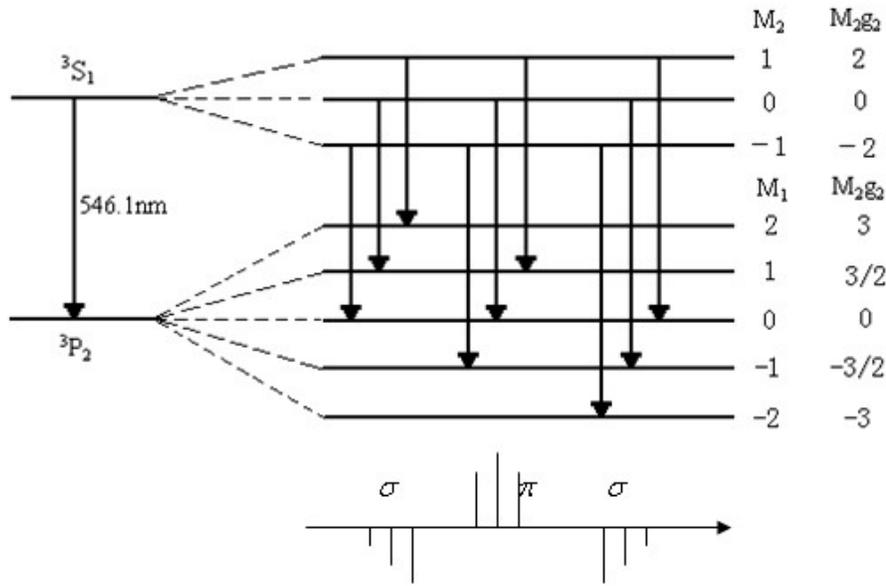


Figure 2 Zeeman effect and intensity distribution of Mercury green line

As seen in the diagram, the upper and lower energy levels are split into 3 and 5 sub levels in an external magnetic field, respectively. The allowed nine transitions by selection rules are shown in the energy level diagram. The appropriate spectral locations of these spectral lines are drawn at the bottom of the energy level diagram with the wave number increasing from left to right equidistantly. The heights of these line segments represent the relative intensities of the actual spectral lines.

5) Theory of Fabry-Perot etalon

As the wavelength difference of Zeeman splitting is very small, a regular prism or grating does not have enough resolution to separate these spectral lines. In this experiment, a Fabry-Perot etalon is used to resolve Zeeman spectral line separations. The working principle of a F-P etalon is as follows.

When a ray of light passes through a plane-parallel plate with two reflecting surfaces, it is reflected many times between the two surfaces and hence multiple-beam interference occurs. The higher the surface reflectance is, the sharper the interference fringes are. That is the basic principle of a Fabry-Perot interferometer. As shown in Figure 3, two partially reflecting mirrors G_1 and G_2 are aligned parallel to each other, forming a reflective cavity. When monochromatic light is incident on the reflective cavity with an angle θ , many parallel rays pass through the cavity to get transmitted. The optical path difference between two neighboring rays is given by δ , as:

$$\delta = 2nd \cos \theta \quad (14)$$

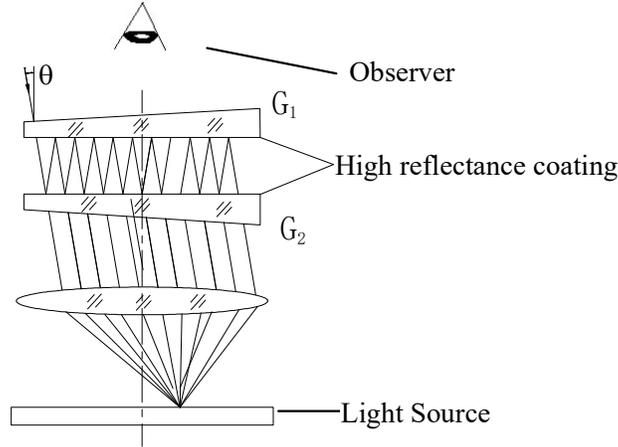


Figure 3 Schematic of Fabry-Perot interferometer

Thus, the transmitted light intensity is:

$$I' = I_0 \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 \frac{\pi\delta}{\lambda}} \quad (15)$$

where I_0 is the incident light intensity, R is the mirror reflectance, n is the refractive index of the medium in the cavity, d is the cavity length or mirror spacing, and λ is the wavelength of the monochromatic light in vacuum.

Thus, I' varies with δ . When

$$\delta = 2nd \cos \theta = m\lambda \quad (m = 0, 1, 2\dots) \quad (16)$$

I' becomes maximum so that constructive interference of the transmitted light occurs.

Since the interference of an etalon is multiple-beam interference, the width of interference pattern becomes very fine (sharp). Usually, the resolution of an etalon is represented by the parameter of finesse F :

$$F = \frac{\pi\sqrt{R}}{1-R} \quad (17)$$

Considering two monochromatic light beams at wavelengths λ_1 and λ_2 with a small wavelength separation ($\lambda_1 > \lambda_2$ and $\lambda_1 \cong \lambda_2 \cong \lambda$), e.g. the split light beams of a Mercury green line by Zeeman effect. For the same order of the interference m , as described in (16), the intensity Maxima of λ_1 and λ_2 correspond to different incident angles θ_1 and θ_2 , forming two sets of interference patterns.

By increasing the wavelength separation (i.e. increasing magnetic field intensity), so the m^{th} order maximum of λ_2 overlaps with the $(m-1)^{\text{th}}$ order maximum of λ_1 , as

$$m\lambda_2 = (m-1)\lambda_1 \quad (18)$$

Under paraxial conditions ($\theta \cong 0$), (16) can be rewritten as $m=2d/\lambda$, thus (18) can be simplified as:

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2d} \quad (19)$$

Represented by wave number, (19) becomes:

$$\Delta\tilde{\nu} = \frac{1}{2d} \quad (20)$$

The calculated $\Delta\lambda$ or $\Delta\tilde{\nu}$ based on (19) or (20) is called the free spectral range of the etalon.

6) Measurement of wavelength separation

By imaging the interference pattern of a F-P etalon to the focal plane of a lens with focal length f , as seen in Fig. 4, The relationship between incident angle θ and diameter D of an interference ring at the central portion of the pattern can be written as:

$$\cos\theta = \frac{f}{\sqrt{f^2 + (D/2)^2}} \approx 1 - \frac{1}{8} \frac{D^2}{f^2} \quad (21)$$

Substitute (21) into (16), we get:

$$2d\left(1 - \frac{D^2}{8f^2}\right) = m\lambda \quad (22)$$

It is apparent from (22) that the square of the diameter of a fringe in central portion has a linear relationship with the order of the interference m . The fringes at a fixed wavelength get denser with an increase in fringe diameter. Further, a larger diameter fringe corresponds to a lower order of the interference. Similarly, for the same order of the interference, a larger diameter fringe corresponds to a smaller wavelength.

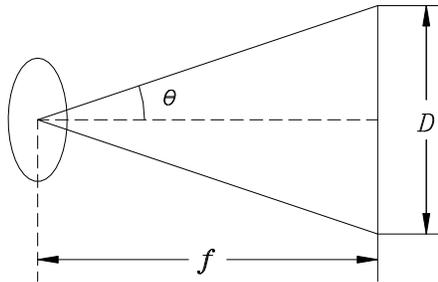


Figure 4 Relationship between incident angle and fringe diameter

The difference between the squares of diameters of adjacent orders of the interference m and $m-1$ at the same wavelength can be derived from (22), as:

$$\Delta D^2 = D_{m-1}^2 - D_m^2 = \frac{4f^2\lambda}{d} \quad (23)$$

Obviously, ΔD^2 is a constant, independent of the order of the interference.

Similarly, the wavelength difference of fringes at the same order of the interference m can be calculated from (22). For example, the wavelength difference between two adjacent spectral

lines from Zeeman splitting can be written as:

$$\lambda_a - \lambda_b = \frac{d}{4f^2 m} (D_b^2 - D_a^2) = \frac{\lambda}{m} \frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} \quad (24)$$

Because the order of the interference m is normally near the central portion, $m \cong 2d/\lambda$. (24) can be rewritten as:

$$\lambda_a - \lambda_b = \frac{\lambda^2}{2d} \frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} \quad (25)$$

Or in wave number:

$$\tilde{\nu}_a - \tilde{\nu}_b = \frac{1}{2d} \frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} = \frac{1}{2d} \frac{\Delta D_{ab}^2}{\Delta D^2} \quad (26)$$

Substituting (26) into (13), we get the charge-mass ratio of an electron:

$$\frac{e}{m} = \frac{2\pi c}{(M_2 g_2 - M_1 g_1) B d} \left(\frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} \right) \quad (27)$$