2. Theory

1) Relationship between total magnetic moment and total angular momentum

Strictly speaking, the total magnetic moment of an atom consists of electron magnetic moment and nuclear magnetic moment. Since the latter is at least three orders smaller than the former, only the electron magnetic moment is considered here. The orbital motion of an electron in an atom creates orbital magnetic moment, while the spin motion of an electron results in spin magnetic moment.

In quantum mechanics, the numerical relationship between the orbital magnetic moment $\boldsymbol{\mu_L}$ and the orbital angular momentum $\boldsymbol{P_L}$ of an electron is as follows:

$$\mu_L = \frac{e}{2m} P_L \text{, with } P_L = \sqrt{L(L+1)}\hbar \tag{1}$$

While the numerical relationship between the spin magnetic moment $\boldsymbol{\mu_S}$ and the spin angular momentum $\boldsymbol{P_S}$ is:

$$\mu_S = \frac{e}{m} P_S \text{, with } P_S = \sqrt{S(S+1)}\hbar \tag{2}$$

where e and m are the charge and the mass of an electron, respectively; L and S represent the orbital and spin quantum numbers of an atom, respectively. The total angular momentum $\boldsymbol{P_J}$ is the sum of the orbital angular momentum and spin angular momentum. The total magnetic moment $\boldsymbol{\mu}$ is the sum of the orbital magnetic moment and spin magnetic moment. As $\boldsymbol{\mu}$ moves around $\boldsymbol{P_J}$ the average effect is not zero in the $\boldsymbol{P_J}$ direction, i.e. only the projection component $\mu_J \neq 0$. The numerical relationship between μ_J and $\boldsymbol{P_J}$ is written as:

$$\mu_J = g \frac{e}{2m} P_J \tag{3}$$

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} \tag{4}$$

where g is called the Lande g-factor, which determines the energy-level splitting amount in the magnetic field.

2) Effect of external magnetic field on atomic energy levels

In an external magnetic field, the introduced torque L on the total magnetic moment μ of an atom is:

$$L = \mu_J \times B \tag{5}$$

where B is the magnetic induction. The torque L forces the angular momentum $\boldsymbol{P_J}$ moving around the magnetic field direction. This motion brings in additional energy:

$$\Delta E = -\mu_J B \cos\alpha \tag{6}$$

Substituting (6) into (3), one gets:

$$\Delta E = g\frac{e}{2m}P_J B\cos\beta \tag{7}$$

Since $\mathbf{P_J}$ is quantized in the direction of the magnetic field, and it must be an integer times of $\hbar$, as,

$$P_J\cos\beta = M\hbar \quad M = J,(J-1),.....,-J \tag{8}$$

There are totally $2J+1$ magnetic quantum numbers. Substituting (8) into (7), one gets:

$$\Delta E = Mg\frac{e\hbar}{2m}B \tag{9}$$

As a result, one energy level is split into 2*J*+1 sub-levels in a magnetic field with the additional energy of each sub-level determined by Eq. (9), which is proportional to the external magnetic field *B* and the Lande *g*-factor.

3) Selection rules of Zeeman effect

Assuming one spectral line is emitted by the electron transition from energy level E_2 to energy level E_1 in the absence of an external magnetic field, the frequency ν of the spectral line is determined by

$$h\nu = E_2 - E_1 \tag{10}$$

By applying an external magnetic field, the upper and lower energy levels are split into $2J_2$+1 and $2J_1$+1 sub-levels with additional energy ΔE_2 and ΔE_1, respectively, which can be derived from Eq. (9). Thus, the frequency of the new spectral line ν' is determined as follows:

$$h\nu' = (E_2 + \Delta E_2) - (E_1 + \Delta E_1) \tag{11}$$

Therefore, the frequency difference between the spectral lines of before and after splitting is:

$$\Delta\nu = \nu' - \nu = \frac{1}{h}(\Delta E_2 - \Delta E_1) = (M_2 - M_1)\frac{eB}{4\pi m} \tag{12}$$

When represented by wave number, the above equation can be rewritten as

$$\Delta\tilde{\upsilon} = (M_2 g_2 - M_1 g_1)\frac{eB}{4\pi mc} \tag{13}$$

If $L=eB/(4\pi mc)$, *L* is called as Lorentz unit. By substituting related physical constants, one gets $L=4.67\times10^{-3}\ Bm^{-1}$, where the unit of *B* is G_S (1 $G_S=10^{-4}$ *T*).

In reality, not all transitions between any energy levels are possible, as transitions must meet certain selection rules as: $\Delta M = M_2\text{-}M_1 = 0, \pm1$ except for $M_2 = 0 \rightarrow M_1 = 0$ when $J_2 = J_1$.

(1) when $\Delta M = 0$, a π line is generated with linear polarization along with the magnetic field when observing in the direction perpendicular to the magnetic field (called transverse Zeeman effect). When observing along the direction parallel to the magnetic field (called longitudinal Zeeman effect), light intensity is zero.

(2) when $\Delta M = \pm1$, $\sigma^{\pm}$ lines are generated. For transverse Zeeman effect, linear polarized light is observed with its polarization perpendicular to the magnetic field; for longitudinal Zeeman

effect, σ^+ line is left-handed circularly polarized light while σ^- line is right-handed circularly polarized light. Since the light source is placed between the two magnetic poles of a magnet, a hole must be opened on one pole in order to observe the longitudinal Zeeman effect.

4) Zeeman effect of Mercury green line

The Mercury green line used in this experiment is at 546.1 nm corresponding to the transition between energy levels $6s7s^3S_1 \rightarrow 6s6p^3P_2$. The two energy levels with the corresponding quantum numbers, g, M , Mg and polarization state, are listed in Tables 1 and 2 below.

Table 1 Polarization states of split lines

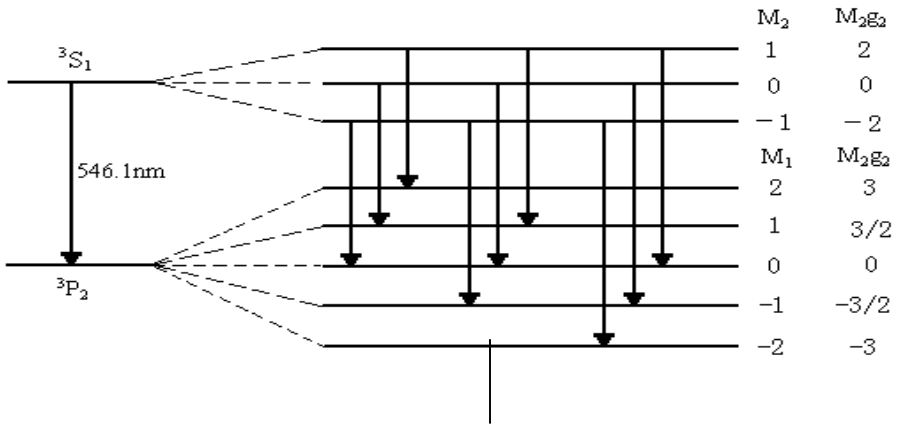
Selection rules	K⊥B (transverse)	K∥B (longitudinal)
△M= 0	Linear polarized π component	No light
△M=＋1	Linear polarized σ component	right-handed circularly polarized
△M=－1	Linear polarized σ component	left-handed circularly polarized

where K is the optical wave vector, B is the magnetic induction vector, σ represents the optical vector E⊥B; π represents the optical vector E∥B.

Table 2 Quantum states with quantum numbers

Atomic states	7^3S_1	6^3P_2
L	0	1
S	1	1
J	1	2
g	2	3/2
M	1, 0, -1	2, 1, 0, -1, -2
Mg	2, 0, -2	3, 3/2, 0, -3/2, -3

The Lande factor g and the splitting in a magnetic field of the two atomic states can be calculated by equations (4) and (7), plotted in transition diagram and shown in Figure 1 below.



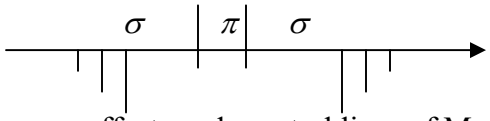


Figure 1 Schematic of Zeeman effects and spectral lines of Mercury green line

It is apparent from Figure 1 that the upper and lower energy levels are split into 3 and 5 sub levels in an external magnetic field, respectively. The allowed nine transitions by selection rules are shown in Figure 1. The appropriate spectral locations of these spectral lines are drawn at the bottom of the energy level diagram with the wave number increasing from left to right equidistantly. The lengths of these lines represent the relative intensities of the spectral lines.

5) Theory of Fabry-Perot Etalon

The wavelength difference of Zeeman splitting is very small, so a regular prism or grating does not have enough resolution power to separate these spectral lines. For this purpose, a Fabry-Perot etalon should be used. The working principle of an F-P etalon is follows.

When a ray of light passes through a plane-parallel plate with two reflecting surfaces, it is reflected many times between the two surfaces and hence multiple-beam interference occurs. The higher the surface reflectance is, the sharper the interference fringes are. That is the basic principle of a Fabry-Perot interferometer. As seen in Figure 2, two partially reflecting mirrors $\mathbf{G_1}$ and $\mathbf{G_2}$ are aligned parallel to each other, forming a reflective cavity. When monochromatic light is incident on the reflective cavity with an angle θ, many parallel rays pass through the cavity to get transmitted. The optical path difference between two neighboring rays is given by

$$\delta = 2nd\cos\theta \tag{14}$$

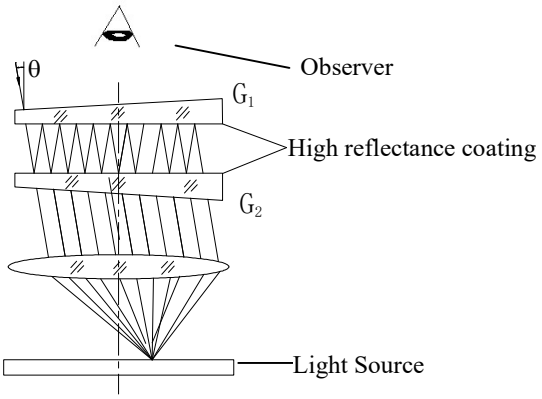


Figure 2 Schematic of Fabry-Perot interferometer

Thus, the transmitted light intensity is:

$$I' = I_0 \frac{1}{1+\dfrac{4R}{(1-R)^2}\sin^2\dfrac{\pi\delta}{\lambda}} \tag{15}$$

where I_0 is the incident light intensity, R is the mirror reflectance, n is the refractive index of the medium in the cavity, d is the cavity length or mirror spacing, and λ is the wavelength of the monochromatic light in vacuum.

Thus, I' varies with δ. When

$$\delta = 2nd\cos(\theta) = m\lambda \quad (m = 0, 1, 2\ldots) \tag{16}$$

I' becomes maximum so that constructive interference of the transmitted light occurs; when

$$\delta = (2m' + 1)\lambda / 2 \qquad (m' = 0, 1, 2\ldots) \tag{17}$$

I' becomes minimum and destructive interference of the transmitted light is observed.

Since the interference of an etalon is multiple-beam interference, the width of interference pattern becomes very fine (sharp). Usually, the resolution of an etalon is represented by the parameter of finesse *F*:

$$F = \frac{\pi\sqrt{R}}{1 - R} \tag{18}$$

When considering two monochromatic light beams λ_1 and λ_2 with small wavelength difference ($\lambda_1 > \lambda_2$ and $\lambda_1 \cong \lambda_2 \cong \lambda$) for the same order of the interference *m*, as described in (16), the intensity Maxima of λ_1 and λ_2 correspond to different incident angles θ_1 and θ_2, forming two sets of interference patterns. By increasing the wavelength separation (i.e. increasing magnetic field intensity), the m^{th} order maximum of λ_2 can overlap with the $(m\text{-}1)^{th}$ order maximum of λ_1, as

$$m\lambda_2 = (m\text{-}1)\lambda_1 \tag{19}$$

Under paraxial conditions ($\theta \cong 0$), (16) can be rewritten as $m = 2d/\lambda$, thus (19) becomes

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2d} \tag{20}$$

Represented by wave number, it is:

$$\Delta\tilde{\upsilon} = \frac{1}{2d} \tag{21}$$

The calculated $\Delta\lambda$ or $\Delta\tilde{\upsilon}$ based on (20) or (21) is called as the free spectral range of the etalon.

6) Measurement of the wavelength difference

If one images the interference pattern of a F-P etalon to the focal plane of a lens with focal length *f*, as seen in Figure 3, the relationship between the incident angle θ and its diameter D of a specified interference ring at the central portion of the pattern can be written as

$$\cos\theta = \frac{f}{\sqrt{f^2 + (D/2)^2}} \approx 1 - \frac{1}{8}\frac{D^2}{f^2} \tag{22}$$

Substitute (22) into (16), we get:

$$2d\left(1 - \frac{D^2}{8f^2}\right) = m\lambda \tag{23}$$

It is apparent from (23) that the square of the diameter of a fringe in central portion has a linear relationship with the order of the interference *m*. The fringes at a fixed wavelength get denser with an increase in fringe diameter. Moreover, a larger diameter fringe corresponds to a lower order of the interference. Similarly, for the same order of the interference, a larger diameter fringe corresponds to a smaller wavelength.

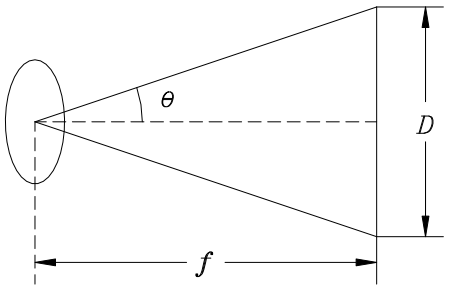


Figure 3 Relationship between incident angle and fringe diameter

The difference between the squares of diameters of adjacent orders of the interference m and m-1 at the same wavelength can be derived from (23), as:

$$\Delta D^2 = D_{m-1}^2 - D_m^2 = \frac{4f^2\lambda}{d} \tag{24}$$

Obviously, ΔD^2 is a constant, independent of the order of the interference.

Similarly, the wavelength difference of fringes at the same order of the interference m can be calculated from (23). For example, the wavelength difference between two adjacent spectral lines from Zeeman splitting can be written as:

$$\lambda_a - \lambda_b = \frac{d}{4f^2 m}(D_b^2 - D_a^2) = \frac{\lambda}{m}\frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} \tag{25}$$

Because the order of the interference m is normally near the central portion, $m \cong 2d/\lambda$, (25) can be rewritten as:

$$\lambda_a - \lambda_b = \frac{\lambda^2}{2d}\frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} \tag{26}$$

Or represented in wave number;

$$\tilde{\upsilon}_a - \tilde{\upsilon}_b = \frac{1}{2d}\frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2} = \frac{1}{2d}\frac{\Delta D_{ab}^2}{\Delta D^2} \tag{27}$$

where $\Delta D_{ab}^2 = D_b^2 - D_a^2$.

Substitute (27) into (13), we get the charge-mass ratio of an electron as:

$$\frac{e}{m} = \frac{2\pi c}{(M_2 g_2 - M_1 g_1)Bd}\left(\frac{D_b^2 - D_a^2}{D_{m-1}^2 - D_m^2}\right) \tag{28}$$